Session 17 *(In preparation for Class 17, students are asked to view Lecture 17.)*

Topics for Class 17

Polyhedron folding: Pita forms, D-forms, seam forms, convex hull and crease properties, rolling belts, Burago-Zalgaller folding into nonconvex polyhedra.

Detailed Description of Class 17

This class focuses on D-forms (introduced by artist Tony Wills) and related constructions called pita forms and seam forms:

- **Pita forms** were demonstrated in Lecture 17: take a convex polygon or curved shape and glue up two halves of its perimeter to make a convex surface.
- **D-forms** extend this to gluing together two convex polygons or curves.
- **Seam forms** further generalize to arbitrarily many, not necessarily convex polygons, but still require Alexandrov's Theorem to apply (actually a smooth version called Alexandrov-Pogorelov).

We'll make physical D-forms and prove two neat properties about them (which originate from a final project in this class from 2007). We'll also briefly review rolling belts, the implementation of Bobenko-Izmestiev's Alexandrov construction, and Burago-Zalgaller's folding of any polygon with any gluing into a nonconvex polyhedron [O'Rourke 2010; Spring 2005].

Topics for Lecture 17

Polyhedron folding: Decision problem, enumeration problem, combinatorial problem, nonconvex solution, convex polyhedral metrics, Alexandrov gluings, Alexandrov's Theorem, Bobenko-Izmestiev constructive proof, pseudopolynomial algorithm, ungluable polygons, perimeter halving, gluing tree, rolling belts.

Detailed Description of Lecture 17

This lecture dives into the problem of folding polygons into polyhedra. The focus here is on folding convex polyhedra, though there is one nice result about the nonconvex case by Burago & Zalgaller.

The main tool in this area is called Alexandrov's Theorem, from 1941, which characterizes when a gluing of the boundary of a polygon will result in a convex polyhedron; plus, as we saw last lecture, that convex result is always unique. We'll sketch a proof of this theorem as well as recent algorithms for finding the convex polyhedron.

With this tool in hand, we'll explore some different properties of gluings. Some polygons, in fact "most" in a certain sense, have no Alexandrov gluings. Convex polygons, on the other hand, always do. Some polygons have infinitely many gluings, but this always happens in a controlled way with a few "rolling belts". Along the way we'll see gluing trees, a useful tool for analyzing gluings that we'll use in the next lecture for algorithms to find gluings.

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
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