6.01: Introduction to EECS I

Circuit Abstractions

April 5, 2011

Midterm Examination #2

Time:	Tuesday, April 12, 7:30 PM to 9:30 PM
Location:	Walker Memorial (if last name starts with A-M)
	10-250 (if last name starts with N-Z)
Coverage:	Everything up to and including Design Lab 9.

You may refer to any printed materials that you bring to exam.

You may use a calculator.

You may not use a computer, phone, or music player.

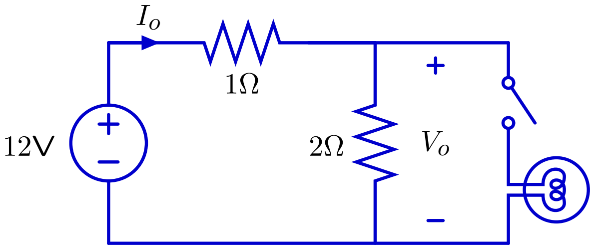
No software lab in week 10.

Review sessions during office hours.

Last Time: Interaction of Circuit Elements

Circuit design is complicated by interactions among the elements. Adding an element changes voltages & currents **throughout** circuit.

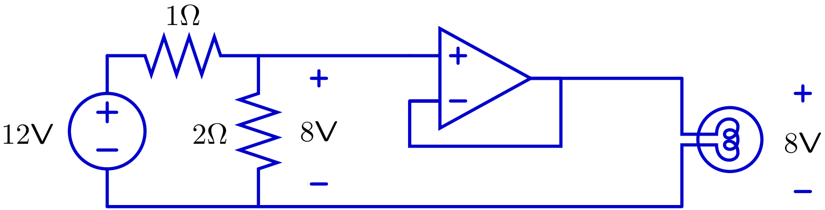
Example: closing a switch is equivalent to adding a new element.



Closing the switch decreases V_o and increases I_o.

Last Time: Buffering with Op-Amps

Buffers can be used to simplify modular design.



This op-amp circuit produces an output voltage equal to its input voltage (8V) while having no effect on the left part of the circuit.

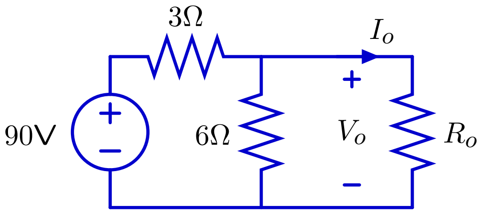
There are also other useful ways to deal with element interactions.

Today: **abstractions** to characterize circuit interactions

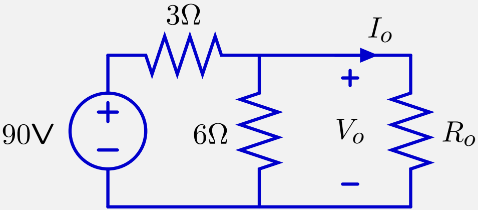
Systematic Changes

Altering an element changes voltages and currents **systematically**.

Example: consider changes in V_o and I_o when R_o is changed.



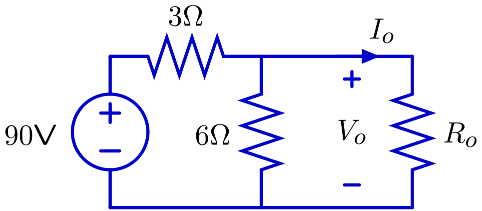
Check Yourself



How many of the blue numbers are wrong?

R_o [Ω]	V_o [V]	I_o [A]
0	0	30
2	30	15
3	36	12
8	48	6
∞	60	0

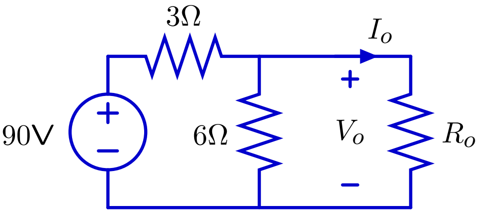
Check Yourself



Two issues (for each value of R_o):

- Does $R_o = \dfrac{V_o}{I_o}$?
- Are KCL and KVL satisfied?

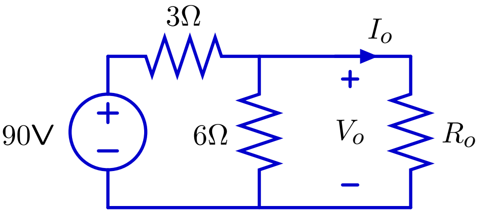
Check Yourself



Does $R_o = \dfrac{V_o}{I_o}$?

R_o [Ω]	V_o [V]	I_o [A]	
0	0	30	√
2	30	15	√
3	36	12	√
8	48	6	√
∞	60	0	√

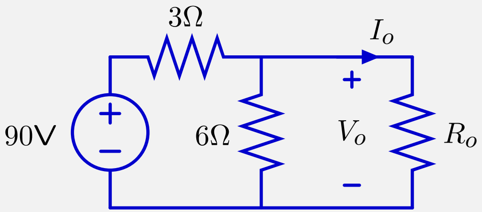
Check Yourself



Are KCL and KVL satisfied? i.e., does $I_o + I_{6\Omega} = I_{3\Omega}$?

R_o [Ω]	V_o [V]	I_o [A]	$I_{6\Omega} = \frac{V_o}{6}$	$I_{3\Omega} = \frac{90-V_o}{3}$	
0	0	30	$\frac{0}{6} = 0$	$\frac{90}{3} = 30$	$\surd$
2	30	15	$\frac{30}{6} = 5$	$\frac{60}{3} = 20$	$\surd$
3	36	12	$\frac{36}{6} = 6$	$\frac{54}{3} = 18$	$\surd$
8	48	6	$\frac{48}{6} = 8$	$\frac{42}{3} = 14$	$\surd$
∞	60	0	$\frac{60}{6} = 10$	$\frac{30}{3} = 10$	$\surd$

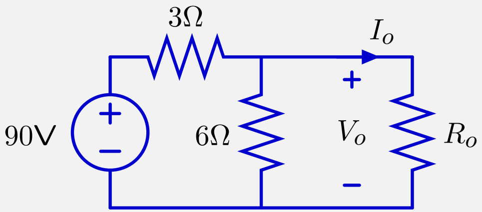
Check Yourself



How many of the blue numbers are wrong? 0

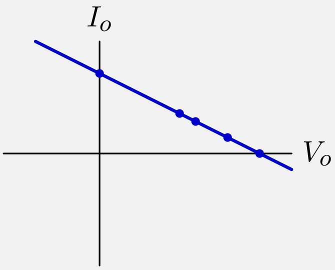
R_o [Ω]	V_o [V]	I_o [A]
0	0	30
2	30	15
3	36	12
8	48	6
∞	60	0

Check Yourself



Interestingly, these numbers fall on a straight line!

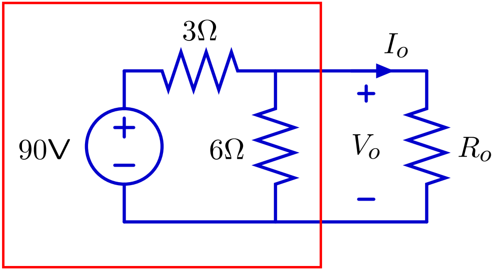
$R_o\,[\Omega]$	V_o [V]	I_o [A]
0	0	30
2	30	15
3	36	12
8	48	6
∞	60	0

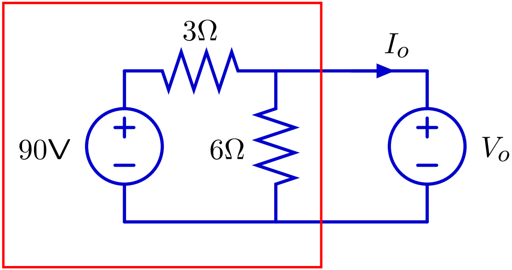


Current-Voltage Relations

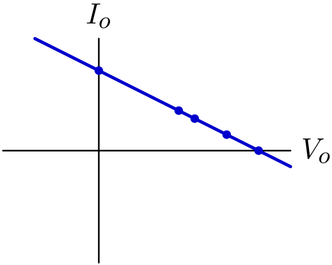
The straight line is a property of the left part of the circuit.

The same current-voltage relation results for **any** circuit to the right!



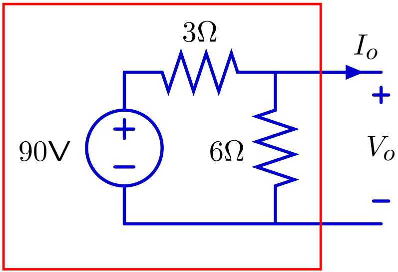


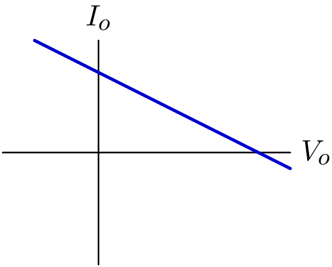
R_o [Ω]	V_o [V]	I_o [A]
0	0	30
2	30	15
3	36	12
8	48	6
∞	60	0



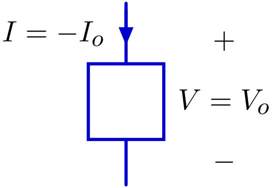
Current-Voltage Relations

The current-voltage relation summarizes **all** possible behaviors of circuit – regardless of what the circuit is connected to.



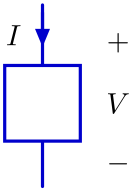


It allows us to think about circuit as a single element: a **one-port**.



One-Ports

A "one-port" is a circuit that is regarded as a single, generalized component.

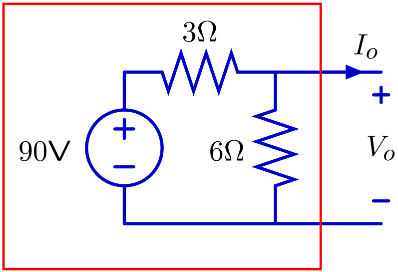


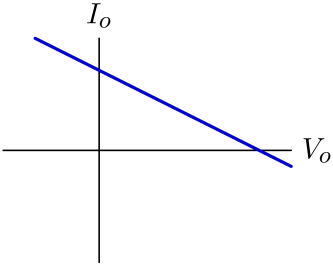
As with other components, a one-port has two terminals:

- current I enters "$+$" terminal and exits "$-$" terminal and
- produces voltage V across the terminals.

Current-Voltage Relations

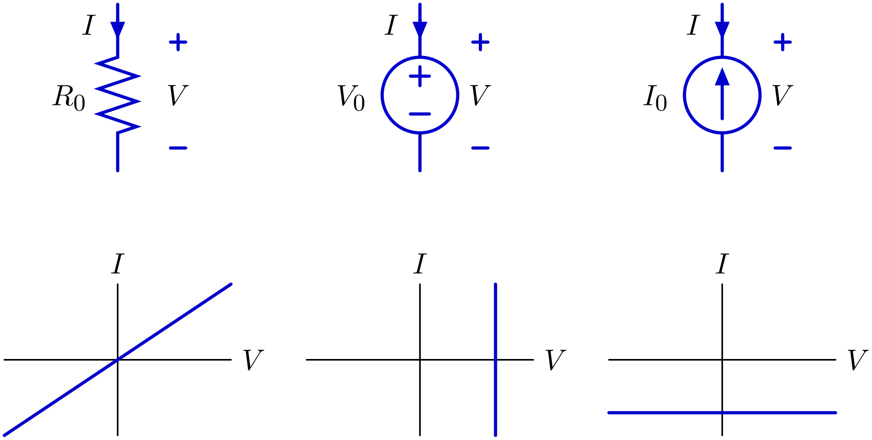
Under what conditions is the current-voltage relation a **straight line**?





Current-Voltage Relations

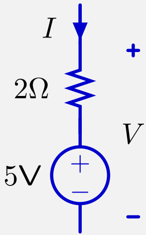
Current-voltage relations for resistors and sources are **straight lines**.



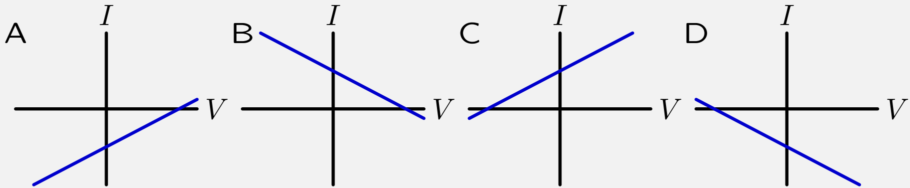
$V = R_0\, I$ $\qquad$ $V = V_0$ $\qquad$ $I = -I_0$

What happens when you combine different types of components?

Check Yourself

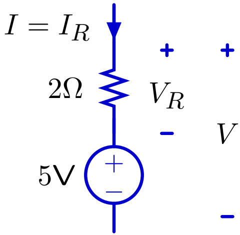


What is the corresponding current-voltage relation?



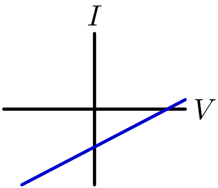
Check Yourself

What is the corresponding current-voltage relation?

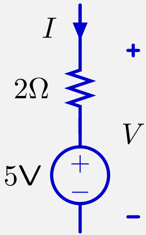


Voltage across resistor: $V_R = V - 5$.

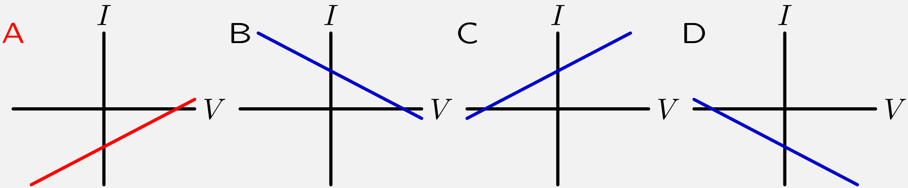
Current through resistor: $I = I_R = \frac{V_R}{R} = \frac{V-5}{2} = \frac{1}{2}V - \frac{5}{2}$.



Check Yourself



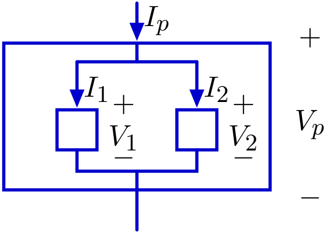
What is the corresponding current-voltage relation? A

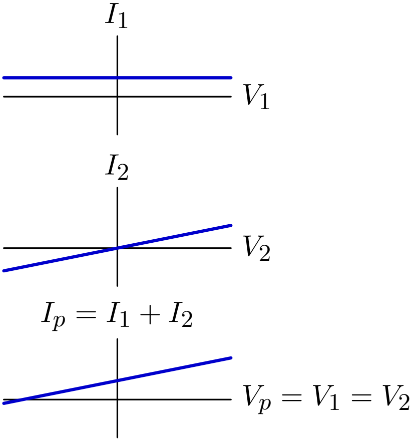


Parallel One-Ports

If the i-v curves for two one-ports are both straight lines, then the i-v curve for the parallel combination is a straight line.

Graphical "proof":



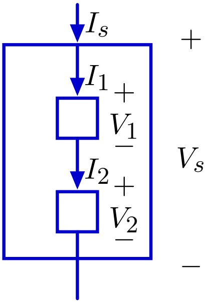


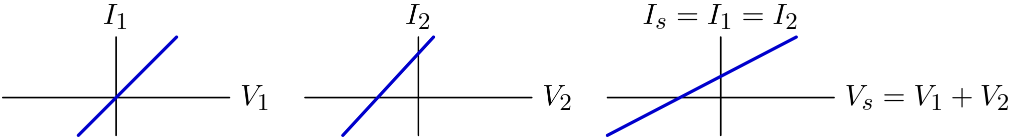
The "sum" of two straight lines is a straight line.

Series One-Ports

If the i-v curves for two one-ports are both straight lines, then the i-v curve for the series combination is a straight line.

Graphical “proof”:



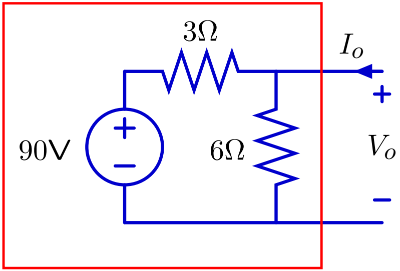


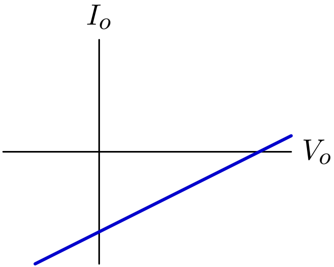
The “horizontal sum” of two straight lines is a straight line.

Current-Voltage Relations

More generally, any combination of one-ports with straight i-v curves will produce a one-port with a straight i-v curve.

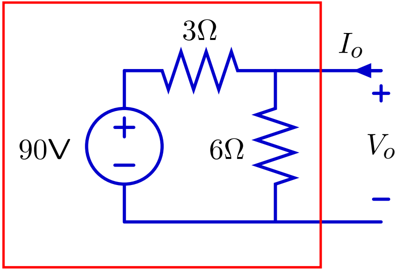
Example: if a one-port contains only resistors and sources, then the current-voltage relation will be a straight line.

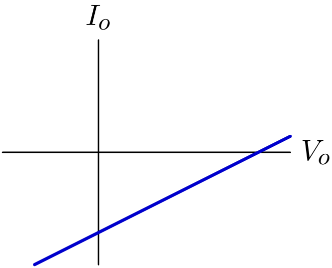




Linear Equations

If the current-voltage relation is a straight line, then the relation between element current and element voltage is a **linear equation**.





Linear equations have the form

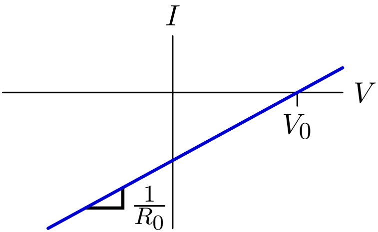
$$a_i V_i + b_i I_i + c_i = 0.$$

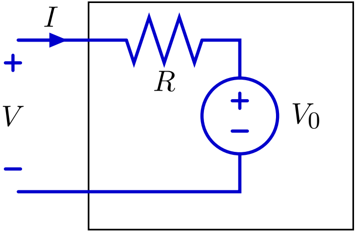
(i.e., there are no V_i^2 terms, no $\sqrt{I_i}$ terms, etc.)

If all of the component equations are linear, then the i-v curve for the one-port will also be linear (since the solution to a system of linear components plus associated KVL and KCL equations is linear).

Thevenin Equivalents

If the current-voltage relation is linear, the current-voltage relation is the same as that for a voltage source in series with a resistor.





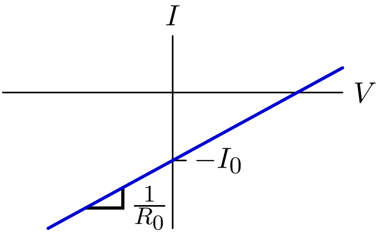
From the circuit, $I = \dfrac{V - V_0}{R}$.

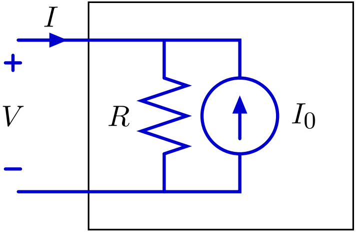
If $I = 0$, then $V = V_0$ (the x-intercept of the plot).

The rate of growth of $I = \dfrac{V}{R} - \dfrac{V_0}{R}$ with V is the slope $1/R$.

Norton Equivalents

If the current-voltage relation is linear, the current-voltage relation is the same as that for a current source in parallel with a resistor.





From the circuit, $V = (I + I_0)R$.

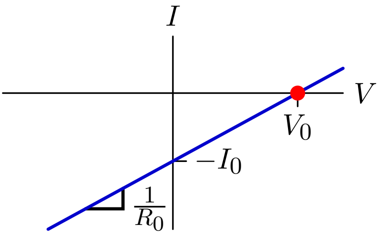
If $V = 0$, then $I = -I_0$ (the negative of the y-intercept of the plot).

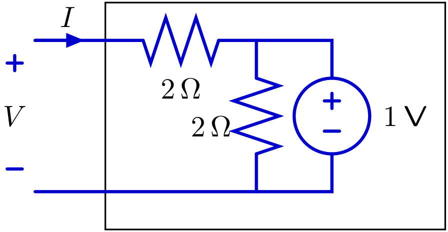
The rate of growth of $I = -I_0 + V/R$ with V is the slope $1/R$.

Open-Circuit Voltage and Short-Circuit Current

If a one-port contains just resistors and current and voltage sources, then its i-v relation is determined by two points.

First point: open-circuit voltage



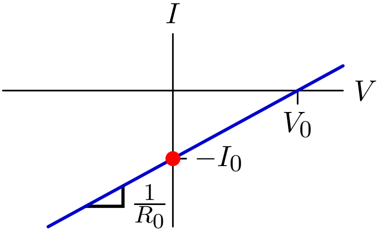


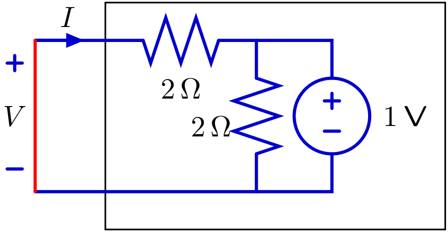
Set $I = 0$ in the circuit. Then $V = V_0 = 1$V.

Open-Circuit Voltage and Short-Circuit Current

If a one-port contains just resistors and current and voltage sources, then its i-v relation is determined by two points.

Second point: short-circuit current



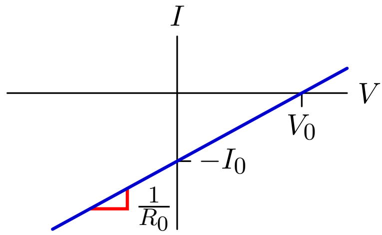


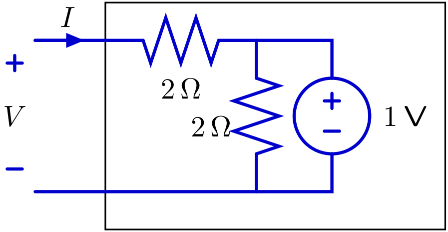
Set $V = 0$ (wire = short circuit). Then $I = -I_0 = -\frac{1}{2}$ A.

Open-Circuit Voltage and Short-Circuit Current

If a one-port contains just resistors and current and voltage sources, then its i-v relation is determined by two points.

The equivalent resistance is $R_0 = \dfrac{V_o}{I_o}$.

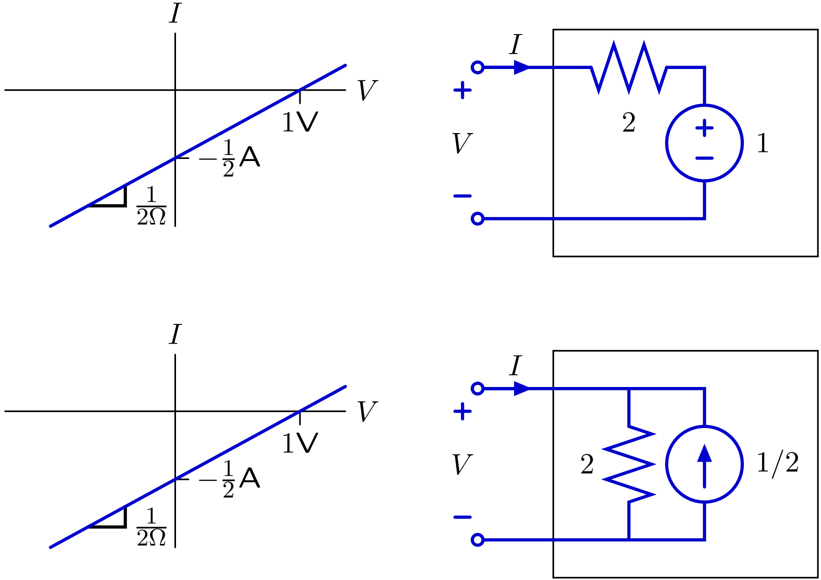




$$R_0 = \frac{V_o}{I_o} = \frac{1\,\mathsf{V}}{\frac{1}{2}\mathsf{A}} = 2\,\Omega$$

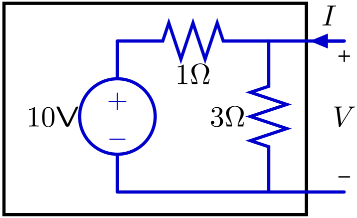
Thevenin and Norton Equivalents

Equivalent circuits.



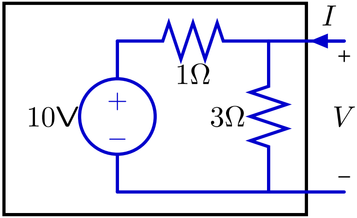
Thevenin Example

Find the Thevenin equivalent of this circuit.



Thevenin Example

Find the Thevenin equivalent of this circuit.

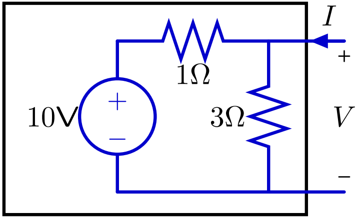


Open-circuit voltage (i.e., $I = 0$)

$$V_0 = V = \frac{3}{3+1} \times 10 = 7.5\,\text{V}$$

Thevenin Example

Find the Thevenin equivalent of this circuit.



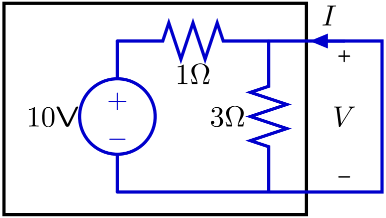
Open-circuit voltage (i.e., $I = 0$)

$$V_0 = V = \frac{3}{3+1} \times 10 = 7.5\,\text{V}$$

Short-circuit current (i.e., $V = 0 \rightarrow$ add a wire!)

Thevenin Example

Find the Thevenin equivalent of this circuit.



Open-circuit voltage (i.e., $I = 0$)

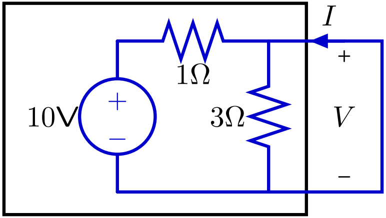
$$V_0 = V = \frac{3}{3+1} \times 10 = 7.5\,\text{V}$$

Short-circuit current (i.e., $V = 0 \rightarrow$ add a wire!)

$$I_0 = -I = \frac{10\,\text{V}}{1\,\Omega} = 10\,\text{A}$$

Thevenin Example

Find the Thevenin equivalent of this circuit.



Open-circuit voltage (i.e., $I = 0$)

$$V_0 = V = \frac{3}{3+1} \times 10 = 7.5\,\mathsf{V}$$

Short-circuit current (i.e., $V = 0 \to$ add a wire!)

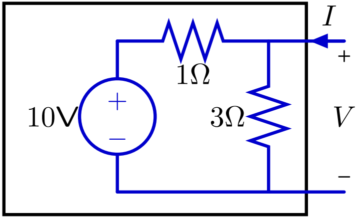
$$I_0 = -I = \frac{10\,\mathsf{V}}{1\,\Omega} = 10\,\mathsf{A}$$

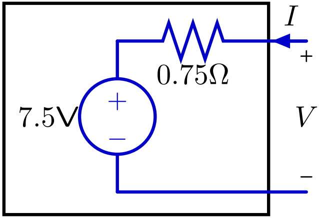
Equivalent resistance

$$R_0 = \frac{V_0}{I_0} = \frac{7.5\,\mathsf{V}}{10\,\mathsf{A}} = 0.75\Omega$$

Thevenin Example

This is the Thevenin equivalent.





Open-circuit voltage (i.e., $I = 0$)

$$V_0 = V = \frac{3}{3+1} \times 10 = 7.5\,\text{V}$$

Short-circuit current (i.e., $V = 0 \rightarrow$ add a wire!)

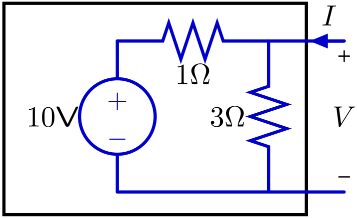
$$I_0 = -I = \frac{10\,\text{V}}{1\,\Omega} = 10\,\text{A}$$

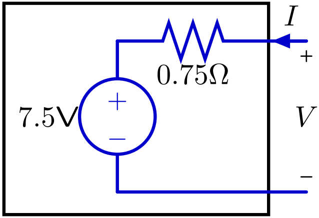
Equivalent resistance

$$R_0 = \frac{V_0}{I_0} = \frac{7.5\,\text{V}}{10\,\text{A}} = 0.75\Omega$$

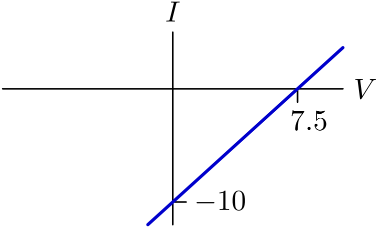
Thevenin Example

This is the Thevenin equivalent.



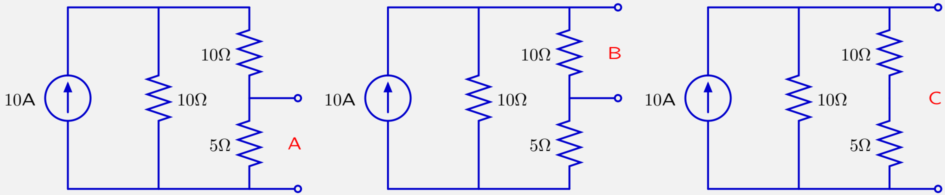


These circuits are "equivalent" in the sense that their i-v curves are the same.



Check Yourself

Determine the equivalent circuits viewed from 3 different ports.

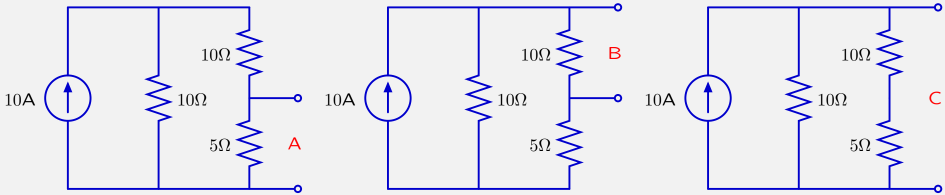


How many of the following are wrong?

	A	B	C
1. $V_0 =$	20V	40V	60V
2. $I_0 =$	5A	5A	10A
3. $R_0 =$	4Ω	6Ω	6Ω

Check Yourself

Determine the equivalent circuits viewed from 3 different ports.



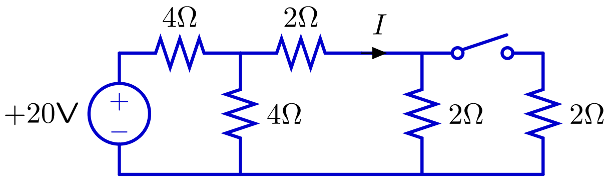
How many of the following are wrong? 1

		A	B	C	
1.	$V_0 =$	20V	40V	60V	
2.	$I_0 =$	5A	5A	10A	should be $\frac{20}{3}$A
3.	$R_0 =$	4Ω	6Ω	6Ω	

Conceptual Value of Equivalent Circuits

Equivalent circuits have conceptual value.

Example: Will closing the switch increase or decrease I?

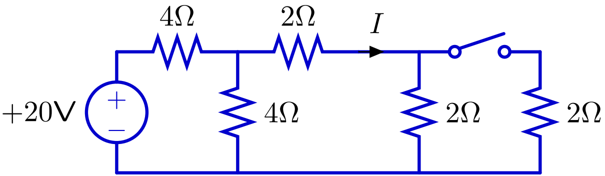


We could just solve two circuits questions – one with switch open and one with switch closed – and compare currents.

But this question can be answered without doing any calculations!

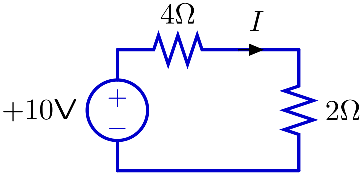
Conceptual Value of Equivalent Circuits

Replace the parts to the left and right of I by equivalent circuits.

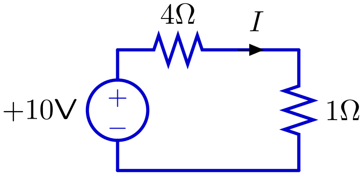


Equivalent circuit to right of I depends on state of switch.

switch open



switch closed

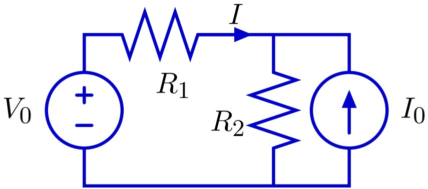


Closing the switch decreases equivalent resistance to right of I.

Therefore, closing the switch increases I.

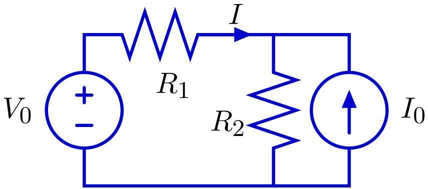
Superposition

If a circuit contains only linear parts (resistors, current and voltage sources), then any voltage (or current) can be computed as the sum of those that result when each source is turned on one-at-a-time.

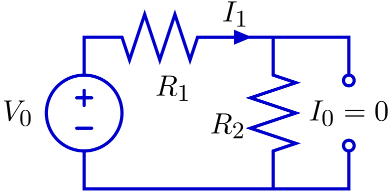


Superposition

If a circuit contains only linear parts (resistors, current and voltage sources), then any voltage (or current) can be computed as the sum of those that result when each source is turned on one-at-a-time.



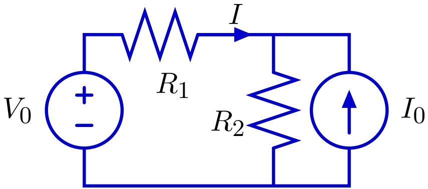
Turning off I_0 (i.e., $I_0 = 0$) is equivalent to an open circuit.



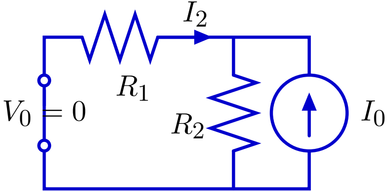
Then $I_1 = \dfrac{V_0}{R_1 + R_2}$.

Superposition

If a circuit contains only linear parts (resistors, current and voltage sources), then any voltage (or current) can be computed as the sum of those that result when each source is turned on one-at-a-time.



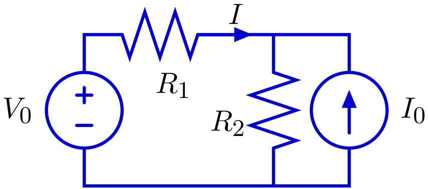
Turning off V_0 (i.e., $V_0 = 0$) is equivalent to a short circuit.



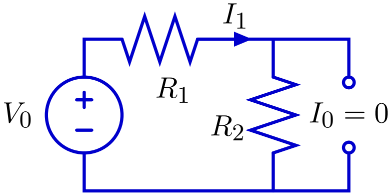
Then $I_2 = -\dfrac{R_2}{R_1 + R_2} I_0$.

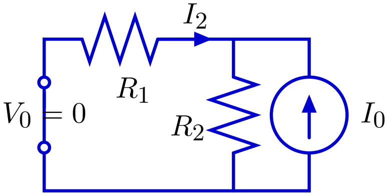
Superposition

If a circuit contains only linear parts (resistors, current and voltage sources), then any voltage (or current) can be computed as the sum of those that result when each source is turned on one-at-a-time.



Combining:

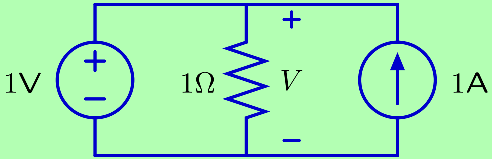




$$I = I_1 + I_2 = \frac{V_0}{R_1 + R_2} - \frac{R_2}{R_1 + R_2} I_0.$$

Check Yourself

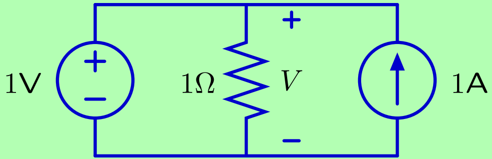
Determine V using superposition.



1. 1V
2. 2V
3. 0V
4. cannot determine
5. none of the above

Check Yourself

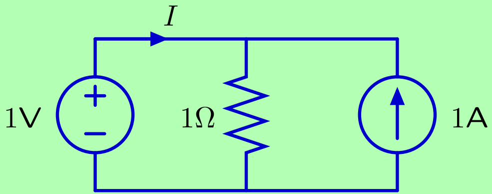
Determine V using superposition. 1



1. 1V
2. 2V
3. 0V
4. cannot determine
5. none of the above

Check Yourself

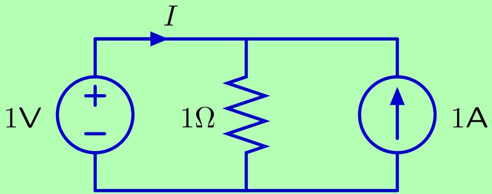
Determine I using superposition.



1. 1A
2. 2A
3. 0A
4. cannot determine
5. none of the above

Check Yourself

Determine I using superposition. 3



1. 1A
2. 2A
3. 0A
4. cannot determine
5. none of the above

Summary

Consequences of linearity.

If a one-port contains just linear elements (resistors, voltage sources, and current sources) then

- the current-voltage relation will be linear, and
- it can be represented by a Thevenin or Norton equivalent circuit.

Linear one-ports can be characterized by two points on their i-v curve (e.g., open-circuit voltage and short-circuit current).

Responses of multiple sources can be superposed to find electrical responses of linear circuits.

This Week

Software lab: Work on HW 3. Good time to get checkoffs.

HW 3: Due at the beginning of design lab.

Design lab: Building one of your circuit designs from HW 3, to make the robot head turn to the light.

Midterm 2:

- Tuesday, April 12, 7:30–9:00PM
- Any printed material okay

Conflict exam:

- Wednesday, April 13, 8:00–9:30AM, 34-501
- Email **`6.01-help@mit.edu`** by Friday April 8 if you need to take this exam.