MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

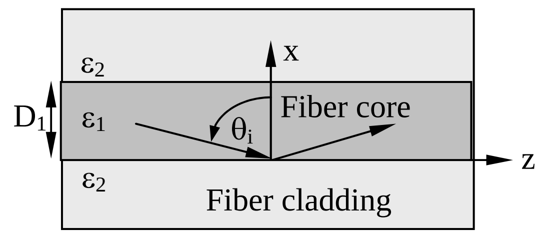
6.013 – Electromagnetics and Applications

Problem Set 12 (six problems)

Suggested Reading: Text: Sections: 12.3-12.4, 13.1-13.2.1, 13.2.3-13.2.4, 13.3, 9.4

Problem 12.1

A certain clad optical fiber can be approximated by a dielectric slab waveguide of thickness D_1 and permittivity $\varepsilon_1 = 4$ that is centered in a dielectric of thickness D_2 and permittivity $\varepsilon_2 = 3.9$, as illustrated. Assume the angle of incidence within the inner fiber is $\theta_i = 85^o$, which is beyond the critical angle θ_c.



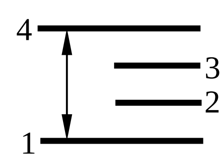
(a) What is the critical angle θ_c?

(b) In terms of the free-space wavelength λ_o, what is the 1/e decay length α^{-1} in the cladding (ε_2), where $\overline{E}(x) = \hat{y} E_o e^{-\alpha x}$? Normally D_2 is made many times thicker than α^{-1} to ensure no external attenuation occurs.

(c) Roughly guess what D_1 is in terms of λ_o for the TE_1 mode. Briefly explain your reasoning.

Problem 12.2

A four-level laser has equally spaced atomic energy levels $E_4 > E_3 > E_2 > E_1$, as illustrated. Assume the spontaneous transition rates A_{41}, A_{42}, A_{31}, and $A_{32} \approx 0$, while the others are larger.



(a) When the 1-4 transition is pumped, in which energy level do most atoms eventually find themselves in the absence of lasing? Briefly explain your reasoning.

(b) For this case, approximately what is the maximum possible energy efficiency of this laser (P_{out}/P_{pump}) for the 3-2 laser transition?

Problem 12.3

A laser diode has $\varepsilon = 4\varepsilon_o$ and a cavity length 1-mm long terminated by one perfect mirror and one mirror with a power transmission coefficient of 1 percent. It operates at 1-micron wavelength and with a natural linewidth of 0.1 percent associated with the width of the energy levels involved ($Q_{\text{natural linewidth}} \approx 10^3$).

(a) What is the frequency spacing (Hz) between resonances for this laser cavity?

(b) Approximately how many different frequency lines are emitted by this laser within its overall natural bandwidth?

(c) What is the approximate width Δf of each of these lines (Hz) associated with the cavity Q_c? A numerical answer is desired, so Q_c must be estimated.

Problem 12.4

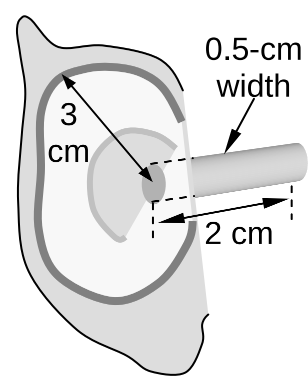
A rectangular acoustic waveguide measures 5×5 millimeters, similar to the ear canal that runs between the outer ear and the eardrum.

(a) What are the cutoff frequencies $f_{0\text{-}3}$ for the lowest four acoustic modes for this waveguide? Assume $c_s = 340\ \text{ms}^{-1}$.

(b) What are the two lowest resonant frequencies f_{001} and f_{002} for the resonator bounded by the eardrum and outer ear for an ear canal that is 2-cm long.

(c) Discuss briefly (guess if necessary) how and why the frequencies found in parts (a) and (b) may relate to the nominal cutoff for human hearing, which is typically 12-20 kHz.

(d) The outer human ear has a ridge that helps focus on the ear canal sounds arriving from the direction in which that person is facing. If a single ridge is approximately three centimeters from the ear canal, as illustrated, what frequencies are favored for this forward focusing effect? What audible frequencies are partially nulled? Note that there are two ridges in most ears, and their spacing is a function of direction. Explain briefly how this could help our cavemen ancestors determine the direction of an unseen predator.



(e) There is a substantial acoustic impedance discontinuity at the junction of the ear canal and the outer ear characterized by the power reflection coefficient $|\underline{\Gamma}|^2$ at the junction. What is the approximate external Q_E of the lowest frequency mode for this ear-canal acoustic resonator in terms of $|\underline{\Gamma}|^2$? Note that the power lost to the resonator (ear canal) via radiation corresponds to power dissipated in the expression for Q, where $Q_E = Q_L$ here.

Problem 12.5

The velocity of sound in the atmosphere is a function of temperature, and has a nominal value of $c_s \cong 340\ \mathrm{ms}^{-1}$. Although air temperatures typically drop with altitude ~7K km^{-1}, there can be temperature inversions for which the air temperature actually increases with altitude, such as when a warm air mass moves in over a cold surface like that of a lake. The acoustic impedance for this problem is $\eta_s \cong 425$.

(a) If 100 watts of 1-kHz acoustic power were confined as a plane wave within 1 square meter, what would be the peak acoustic pressure $|\underline{p}|$ associated with it?

(b) What would be the peak acoustic velocity $|\underline{u}|$?

(c) What is the peak-to-peak motion D [m] of the air molecules in this wave?

(d) If this wave were travelling upward and were incident on a warmer air mass with $c_{s,warm}$ one percent greater than c_s for the cooler air mass below, what would be the critical angle θ_c for this acoustic wave?

(e) If the angle of incidence on this warmer air mass equals θ_c so that an evanescent acoustic wave is produced above the interface that decays as $e^{-\alpha z}$, where z is altitude, what is the value of α in this singular case?

Problem 12.6

A typical closed door consists of a thin massive planar sheet blocking the acoustic path. Assume its density $\rho_d = 1000\ \mathrm{kg/m^3}$ while that of air is $\sim\rho_o \cong 1\ \mathrm{kg/m^3}$. The corresponding velocities of sound c_s are 1050 and 330 ms^{-1}, respectively. Assume normal incidence for all acoustic waves.

(a) What is the ratio of the characteristic acoustic impedances η_o/η_d for the air and the door? ($\eta = \rho c_s$ is analogous to Z_o.)

(b) If the door were infinitely thick, what fraction of the incident acoustic power would the door reflect?

(c) What is the lowest acoustic frequency f_{pass} (if any) that could pass through this ideal lossless 5-cm thick door without any attenuation whatever?

(d) This 5-cm thick door has maximum reflectivity $|\underline{\Gamma}|^2$ at certain frequencies, i.e. its maximum power to stop sound; what is the lowest such frequency f_{stop} and what is the corresponding $|\underline{\Gamma}|^2$?