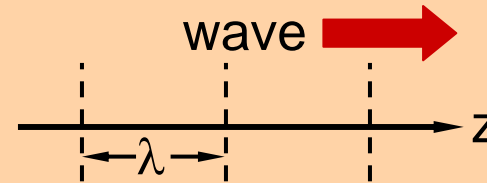


UNIFORM PLANE WAVES

Z-Directed Wave:

$$\underline{\underline{E}} = \underline{\underline{E}}_0 e^{-jkz} = \underline{\underline{E}}_0 e^{-j \underbrace{2\pi z / \lambda}_{\phi}}$$

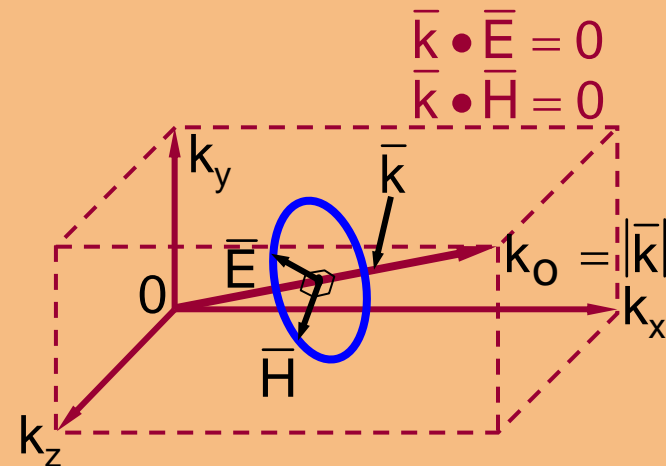
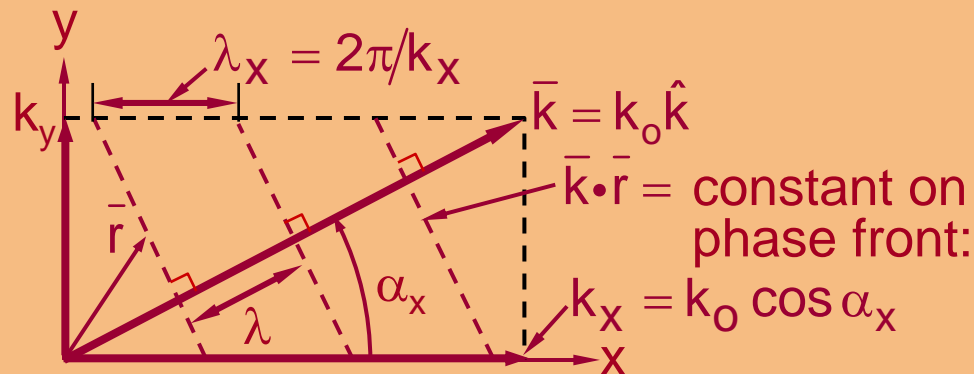
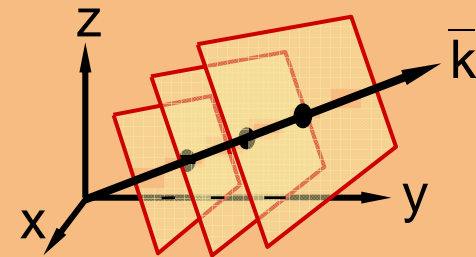


“Phase fronts”

3-D Wave:

$$\underline{\underline{E}} = \underline{\underline{E}}_0 e^{-jk_x x - jk_y y - jk_z z}$$

$$\underline{\underline{E}} = \underline{\underline{E}}_0 e^{-j\bar{k} \cdot \bar{r}} \quad \text{where} \quad \begin{cases} \bar{k} \triangleq \hat{x}k_x + \hat{y}k_y + \hat{z}k_z \\ \bar{r} \triangleq \hat{x}x + \hat{y}y + \hat{z}z \end{cases}$$



PLANE WAVES PROPAGATING AT ANGLES

Dispersion Relation:

Substitute $\bar{\mathbf{E}}_0 e^{-j\bar{\mathbf{k}} \cdot \bar{\mathbf{r}}}$ into wave equation $(\nabla^2 + k_0^2)\bar{\mathbf{E}} = 0$:

$$\left. \begin{array}{l} \bar{\mathbf{E}}_0 e^{-j\bar{\mathbf{k}} \cdot \bar{\mathbf{r}}} = \bar{\mathbf{E}}_0 e^{-j(k_x x + k_y y + k_z z)} \\ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \end{array} \right\} \Rightarrow k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon \triangleq k_0^2$$

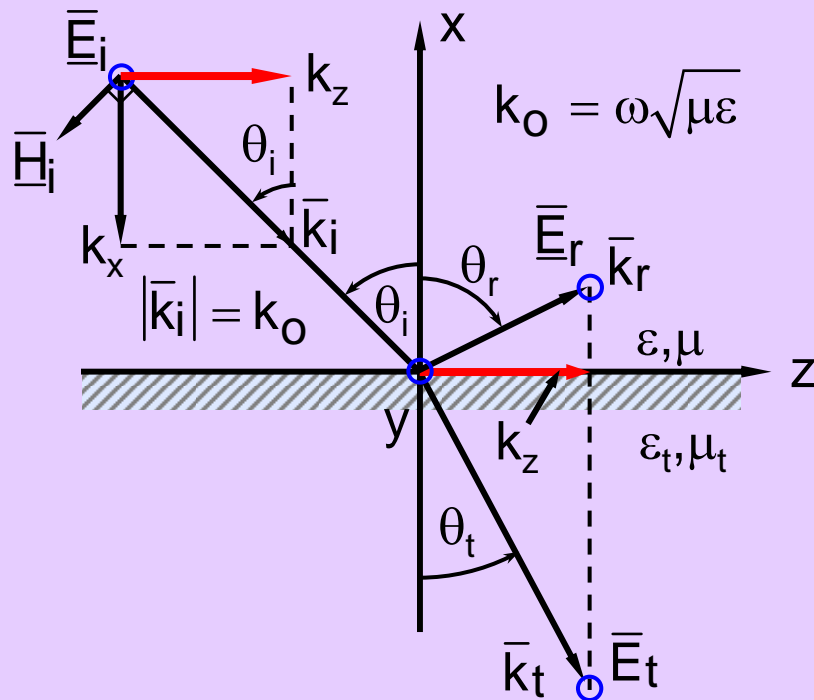
Wave Vector $\bar{\mathbf{k}}$:

Perpendicular to uniform plane wave phase front,

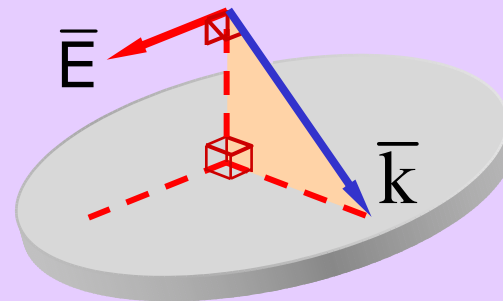
Therefore perpendicular to $\bar{\mathbf{E}}$ and $\bar{\mathbf{H}}$

UPW AT PLANAR BOUNDARY

Case I: TE Wave



“Transverse Electric”
 $\triangleq \bar{E} \perp$ Plane of incidence



Trial Solutions:

Incident: $\bar{E}_i = \hat{y}E_0 e^{+jk_x x - jk_z z} = \hat{y}E_0 e^{+j(k_0 \cos\theta_i)x - j(k_0 \sin\theta_i)z}$

Reflected: $\bar{E}_r = \hat{y}\Gamma E_0 e^{-j(k_0 \cos\theta_r)x - j(k_0 \sin\theta_r)z}$

Transmitted: $\bar{E}_t = \hat{y}\mathcal{T} E_0 e^{+j(k_t \cos\theta_t)x - j(k_t \sin\theta_t)z}$

IMPOSE BOUNDARY CONDITIONS

$\bar{E}_{//}$ is continuous at $x = 0$:

$$E_o e^{-jk_o \sin \theta_i z} + \underline{\Gamma} E_o e^{-jk_o \sin \theta_r z} = \underline{\Gamma} E_o e^{-jk_t \sin \theta_t z} \quad \text{for all } z$$

$$\text{Therefore: } \begin{cases} \underbrace{k_o \sin \theta_i}_{k_{iz}} = \underbrace{k_o \sin \theta_r}_{k_{rz}} = \underbrace{k_t \sin \theta_t}_{k_{tz}} = k_z \\ \theta_r = \theta_i \quad \text{Angle of incidence equals angle of reflection} \end{cases}$$

Snell's Law:

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{k_o}{k_t} = \frac{\omega \sqrt{\mu \epsilon}}{\omega \sqrt{\mu_t \epsilon_t}} = \frac{v_t}{v_i} = \frac{n_i}{n_t}$$

"Snell's Law"

$$\text{where } n \triangleq c/v_{\text{phase}} = c\sqrt{\mu \epsilon}$$

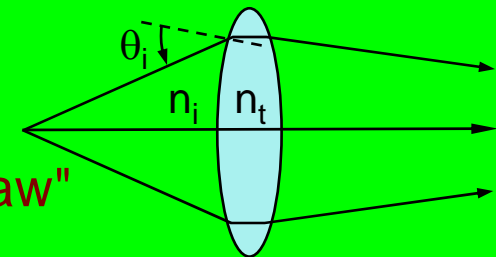
"Refractive Index"

$$n_{\text{vacuum}} = 1$$

$$n_{\text{water}} \cong 1.3 \text{ at visible wavelengths}$$

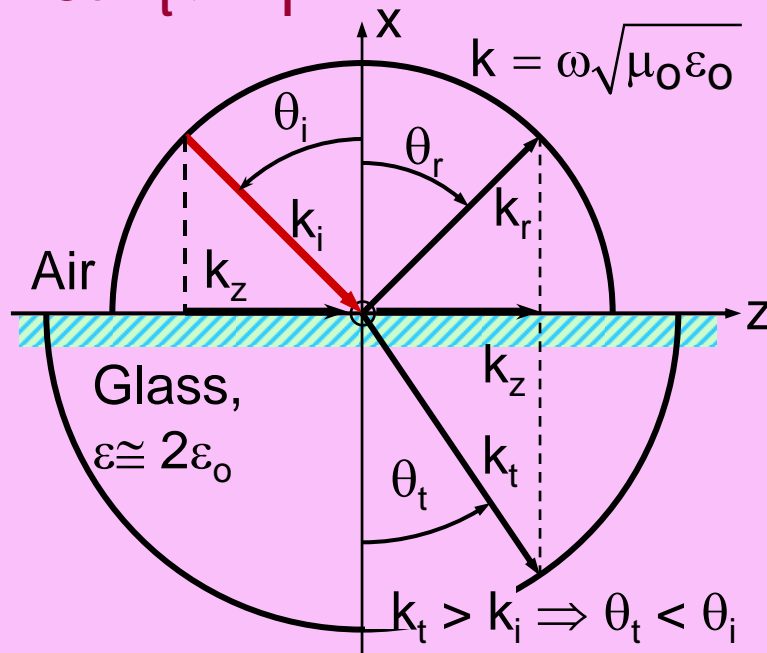
$$n_{\text{glass}} \cong 1.5 - 1.66$$

$$\cong 9 \text{ at audio-radio frequencies}$$

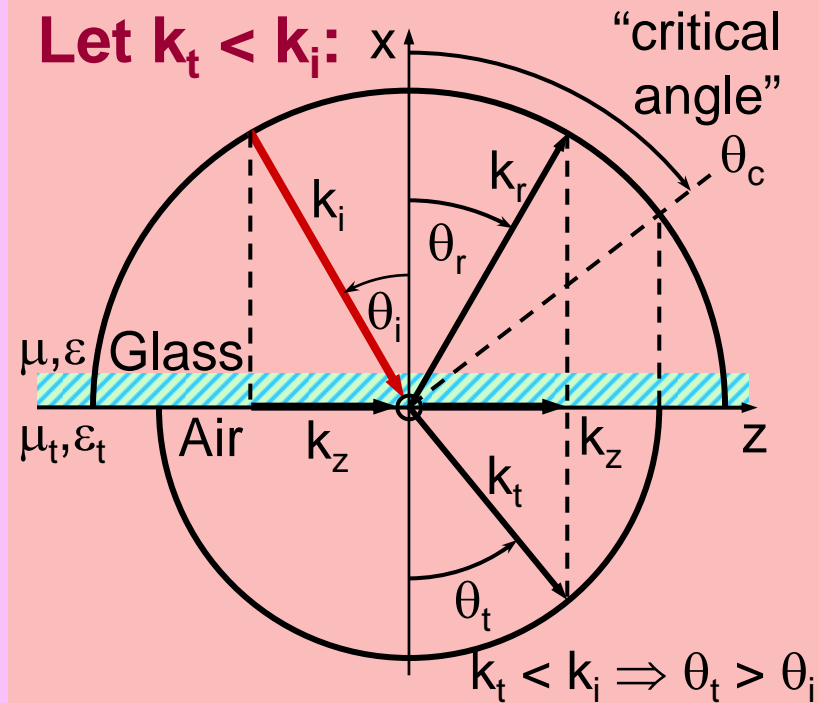


SNELL'S LAW: $k_o \sin \theta_i = k_t \sin \theta_t$

Let $k_t > k_i$:



Let $k_t < k_i$:



Critical Angle θ_c :

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{n_i}{n_t}$$

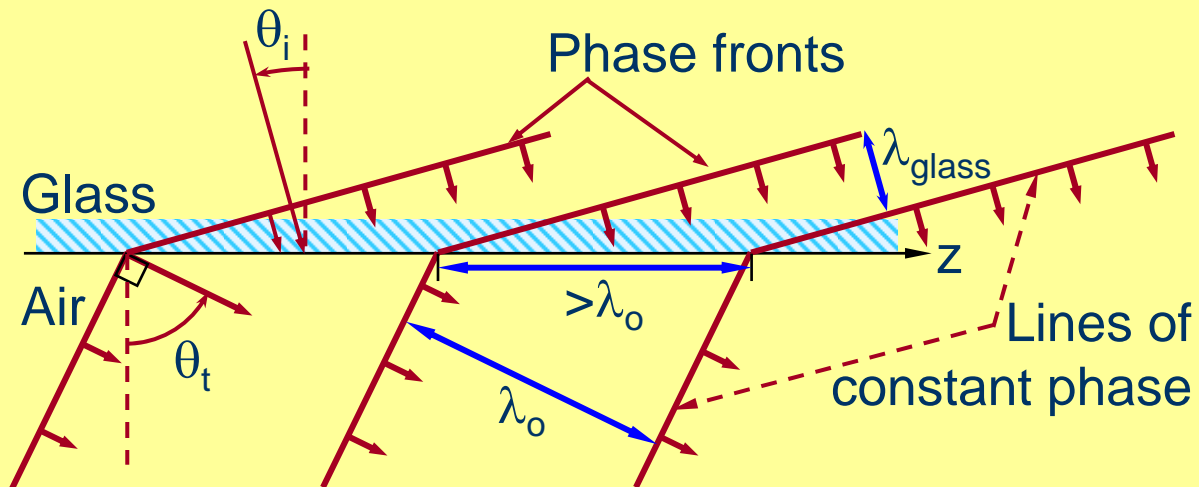
Therefore: $\theta_t \rightarrow 90^\circ \Rightarrow \sin \theta_t \rightarrow 1$ as $\theta_i \rightarrow \theta_c$

$$\theta_c = \sin^{-1}(n_t/n_i) \text{ "critical angle"}$$

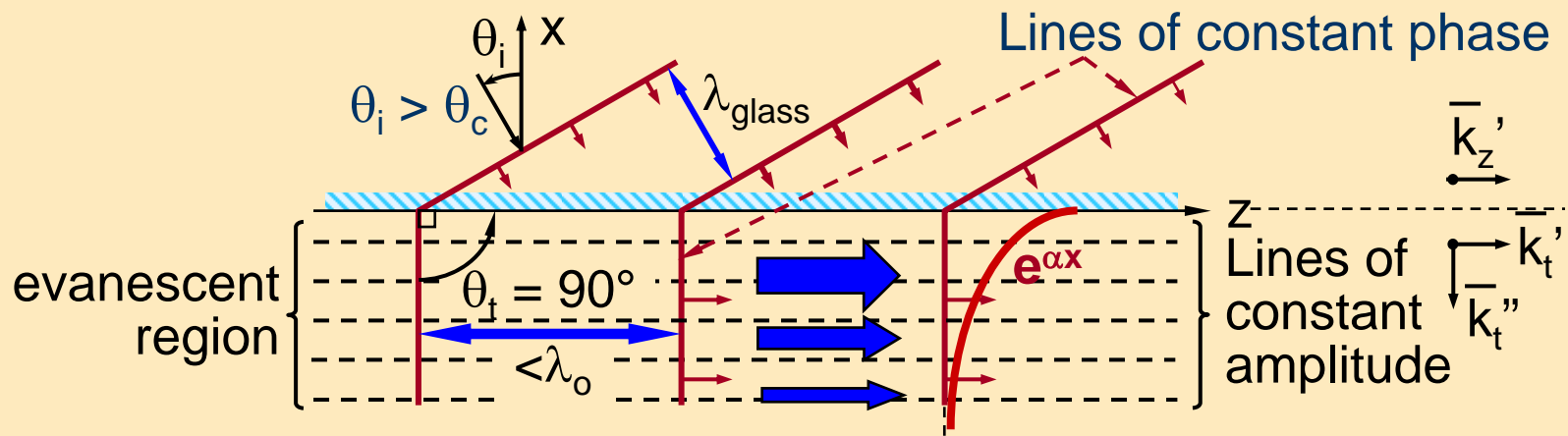
$$\text{e.g., } [\epsilon_i = 2\epsilon_0] \Rightarrow [n_i = \sqrt{2}] \Rightarrow [\theta_c = 45^\circ]$$

NON-UNIFORM PLANE WAVES (NUPW)

Normal refraction: $\theta_i < \theta_c$



Beyond the critical angle, evanescence:



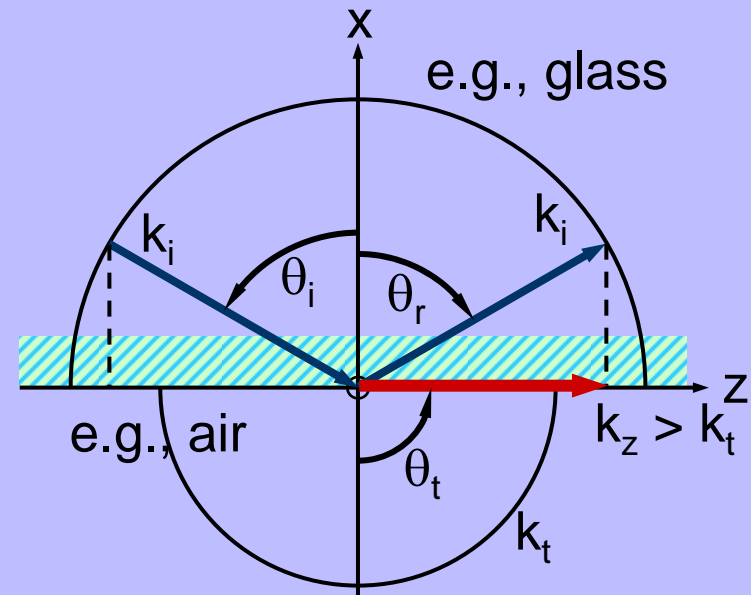
EVANESCENT WAVES

k_{tx} when $\theta > \theta_c$:

Since: $k_t^2 = k_z^2 + k_{tx}^2 = \omega^2 \mu_t \epsilon_t$

Therefore:
$$\begin{cases} k_{tx}^2 = k_t^2 - k_z^2 < 0 \\ k_{tx} = \pm j\alpha = \pm j\sqrt{k_z^2 - k_t^2} \end{cases}$$

Where: $k_z^2 = \omega^2 \mu_i \epsilon_i \sin^2 \theta_i$



Fields when $\theta > \theta_c$:

$$\bar{E}_t = \hat{y} \Gamma E_0 e^{-jk_z z + jk_{tx} x} = \hat{y} \Gamma E_0 e^{+\alpha x - jk_z z} \quad (x < 0)$$

More generally: $\bar{E}_t = \hat{y} \Gamma E_0 e^{-j\bar{k}_t \cdot \bar{r}}$

where: $\bar{k}_t = k_z \hat{z} - j\alpha \hat{x} \triangleq \bar{k}' - j\bar{k}''$

If lossless medium, $\bar{k}' \cdot \bar{k}'' = 0$ $\bar{E}, \bar{H} \propto e^{-j(\bar{k}' - j\bar{k}'') \cdot \bar{r}}$

EVANESCENT WAVES -- SUMMARY

Names:

“non-uniform plane wave”

“evanescent wave” ($\langle S(t) \rangle = 0$ in direction of decay)

“surface wave”

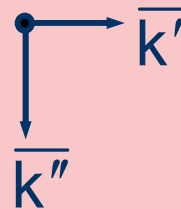
“inhomogeneous plane wave”

Lossless medium:

$$\underline{\bar{E}}, \underline{\bar{H}} \propto e^{-j(\bar{k}' - j\bar{k}'') \cdot \bar{r}}$$

$$\bar{k}' \cdot \bar{k}'' = 0$$

$$k_{tx} = \pm j\alpha = \pm j\sqrt{k_z^2 - k_t^2}$$



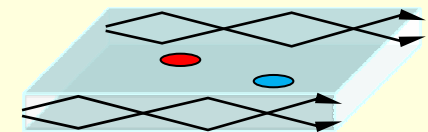
Lossy medium:

$$\bar{k}' \cdot \bar{k}'' \neq 0$$

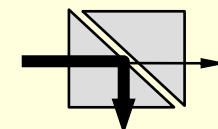
Applications:



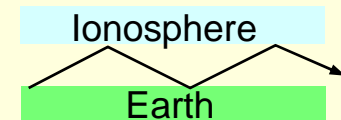
Optical fibers



Flat panel displays



Variable couplers



Global communications

MIT OpenCourseWare
<http://ocw.mit.edu>

6.013 Electromagnetics and Applications
Spring 2009

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.