PRACTICE FINAL
MATH 18.703, MIT, SPRING 13

You have three hours. This test is closed book, closed notes, no calculators.

There are 11 problems, and the total number of points is 180. Show all your work. *Please make your work as clear and easy to follow as possible.* Points will be awarded on the basis of neatness, the use of complete sentences and the correct presentation of a logical argument.

Name:______________________________

Signature:______________________________

Student ID #:______________________________

Problem	Points	Score
1	30	
2	15	
3	15	
4	15	
5	15	
6	15	
7	10	
8	10	
9	15	
10	10	
11	25	
Presentation	5	
Total	180	

1. (30pts) Give the definition of a group.

(ii) Give the definition of an automorphism of groups.

(iii) Give the definition of D_n, the dihedral group.

(iv) Give the definition of an ideal.

(v) Give the definition of a principal ideal domain.

(vi) Give the definition of a unique factorisation domain.

2. (15pts)
(i) Let G be a group and let $\sim$ be the relation $g_1 \sim g_2$ if there is an element $h \in G$ such that $g_1 = hg_2h^{-1}$. Show that $\sim$ is an equivalence relation.

(ii) If $G = S_5$ then identify the equivalence classes.

3. (15pts) Classify all groups of order at most ten.

4. (15pts) (i) State the second isomorphism theorem.

(ii) Prove the second isomorphism theorem.

5. (15pts) (i) State Sylow's theorems.

(ii) Let G be a group of order pqr, where p, q and r are distinct primes. Show that G is not simple.

6. (15pts) (i) If the prime ideal P contains the product IJ of two ideals then prove that P contains either I or J.

(ii) Exhibit a natural bijection between the prime ideals of R/IJ and $R/I \cap J$.

(iii) Give an example of a ring R, and ideals I and J such that IJ and $I \cap J$ are different.

7. (10pts) Does every UFD R, which is not a field, contain infinitely many irreducible elements which are pairwise not associates? If your answer is yes then prove it and if no then give an example.

8. (10pts) Give an example of an integral domain such that every element of R can be factored into irreducibles and yet R is not a UFD.

9. (15pts) (i) Show that $\mathbb{Z}[i]$ is a Euclidean domain.

(ii) Is $6 - i$ prime in $\mathbb{Z}[i]$?

10. (10pts) Write down all irreducible polynomials of degree 2 over the field $\mathbb{F}_5[x]$.

11. (25pts) (i) State Gauss' Lemma and Eisenstein's criteria.

(ii) Show that the polynomial $1 + x^3 + x^6 \in \mathbb{Q}[x]$ is irreducible (**Hint: try a substitution**.)

(iii) Show that the polynomial $1 - t^2 + t^5$ is irreducible over $\mathbb{Q}$ (**Hint: consider the ring $\mathbf{F}_2[t]$.**)