Part I Problems

Problem 1: Find the general solution by separation of variables:

$$\frac{dy}{dx} = 2 - y, \qquad y(0) = 0$$

Problem 2: Find the general solution by separation of variables:

$$\frac{dy}{dx} = \frac{(y-1)^2}{(x+1)^2}$$

Problem 3: The rate of change of a certain population is proportional to the square root of its size. Model this situation with a differential equation.

Problem 4:

The rate of change of the velocity of an object is proportional to the square of the velocity. Model this situation with a differential equation.

Problem 5:

In a population of fixed size S, the rate of change of the number N of persons who have heard a rumor is proportional to the number of those who have not yet heard it. Model this situation with a differential equation.

Problem 6: The amount of a certain medicine in the bloodstream decays exponentially with a half-life of 5 hours. In order to keep a patient safe during a one-hour procedure, there needs to be at least 50 mg of medicine per kg of body weight. How much medicine should be administered to a 60kg patient at the start of the procedure?

Problem 7: Early one morning it starts to snow. At 7AM a snowplow sets off to clear the road. By 8AM, it has gone 2 miles. It takes an additional 2 hours for the plow to go another 2 miles. Let $t = 0$ when it begins to snow, let x denote the distance traveled by the plow at time t. Assuming the snowplow clears snow at a constant rate in cubic meters/hour:

a) Find the DE modeling the value of x.

b) When did it start snowing?

Problem 8: A tank holds 100 liters of water which contains 25 grams of salt initially. Pure water then flows into the tank, and salt water flows out of the tank, both at 5 liters/minute. The mixture is kept uniform at all times by stirring.

a) Write down the DE with IC for this situation.

b) How long will it take until only 1 gram of salt remains in the tank?

18.03SC Differential Equations
Fall 2011