18.034 Honors Differential Equations
Spring 2009

18.034 Recitation: February 26th, 2009

1. Under what conditions on b and k do *all* solutions $y(t)$ to

$$y'' + by' + ky = 0$$

   tend to zero as $t \to \infty$? What is the physical significance of these conditions for a spring system?

2. Let u and v be continuous and linearly independent on an interval I. Suppose w is a function on I with only finitely many zeros.

   (a) Show that wu and wv are linearly independent on I.

   (b) You can't use the Wronskian in this problem. Why not?

   (c) Show that the result can fail if u and v are not continuous.

3. Show that e^t, e^{-t}, and e^{2t} are linearly independent on $\mathbb{R}$ without using the Wronskian.

4. Show that a function y satisfying

$$e^x y'' + (\sin x)y' - (1+x)y \geq 0, \quad y(0) \geq 0, y'(0) > 0,$$

   must be strictly increasing.

5. Consider the problem

$$w'' + \lambda q w = 0, \quad w(a) = w(b) = 0$$

   where $\lambda \in \mathbb{R}$ and $q = q(x)$ is a positive function of x. Show that there are no non-trivial solutions if $\lambda < 0$.