Recitation 10, March 9, 2010

Gain and phase lag; resonance; undetermined coefficients

Exponential response formula: A solution to $p(D)x = Ae^{rt}$ is given by $x_p = A\frac{e^{rt}}{p(r)}$, as long as $p(r) \neq 0$.

ERF with resonance: Assume that $p(r) = 0$. A solution to $p(D)x = Ae^{rt}$ is given by $x_p = A\frac{te^{rt}}{p'(r)}$, as long as $p'(r) \neq 0$.

1. Explain the notation in the ERF.

2. Explain why a spring/mass/dashpot system driven through the spring is modeled by the equation $m\ddot{x} + b\dot{x} + kx = ky$. Here x measures the position of the mass, y measures the position of the other end of the spring, and $x = y$ when the spring is relaxed.

3. In this system, regard $y(t)$ as the input signal and $x(t)$ as the system response. Take $m = 1$, $b = 3$, $k = 4$, $y(t) = A\cos t$. Replace the input signal by a complex exponential y_{cx} of which it is the real part, and use the Exponential Response Formula to compute the exponential ("steady state") system response z_p. Compute H such that $z_p = Hy_{cx}$; H is the *complex gain.* Find $|H|$ and ϕ such that $H = |H|e^{-i\phi}$. Use this information to compute the gain and the phase lag of the original system. What is the steady state solution? Is the amplitude of vibration of the mass greater than or less than the amplitude A of the motion of the far end of the spring?

4. Find the polynomial solution of $\ddot{x} - x = t^2 + t + 1$.

5. Find a solution of $\ddot{x} + 4x = \cos(2t)$ by attempting to use the ERF on a complex replacement.

18.03 Differential Equations
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