18.03 Problem Set 7

I encourage collaboration on homework in this course. However, if you do your homework in a group, be sure it works to your advantage rather than against you. Good grades for homework you have not thought through will translate to poor grades on exams. **You must turn in your own writeups of all problems, and, if you do collaborate, you must write on the front of your solution sheet the names of the students you worked with.**

Because the solutions will be available immediately after the problem sets are due, **no extensions will be possible**.

III. Fourier series, Dirac delta function, and Laplace transform

L26	F 9 Apr	Laplace transform: basic properties: EP 4.1.
L27	M 12 Apr	Application to ODEs: SN 20; Notes H; EP 4.2.
R18	T 13 Apr	Laplace transform.
L28	W 14 Apr	Second order equations; completing the square; EP 4.3; SN 20.
R19	Th 15 Apr	Laplace transform and differential equations.
L29	F 16 Apr	The pole diagram: SN 22, 23.
L30	W 21 Apr	Transfer function and complex gain.
R20	Th 22 Apr	Review.
L31	F 23 Apr	**Hour Exam III**

Part I.

26. (F 9 Apr) Let z be a complex number. Find, from the definition, $\mathcal{L}[e^{zt}]$, and find the region of convergence of the integral.

27. (M 12 Apr) Notes 3A-9 (Do (a) using the s-shift rule as suggested, but then do it again by writing $e^{-t}\sin(3t)$ as a linear combination of complex exponentials.); Notes 3B-1; EP 4.3: 1, 5.

28. (W 14 Apr) EP 4.3: 7, 8, 12, 13, 28, 36.

Part II.

26. (F 9 Apr) [Laplace transform] **(a)** Suppose that $F(s)$ is the Laplace transform of $f(t)$, and let $a > 0$. Find a formula for the Laplace transform of $g(t) = f(at)$ in terms of $F(s)$, by using the integral definition and making a change of variable. Verify your formula by using formulas and rules to compute both $\mathcal{L}[f(t)]$ and $\mathcal{L}[f(at)]$ with $f(t) = t^n$.

(b) Use your 18.02 skills: Show that if $h(t) = f(t) * g(t)$ then $H(s) = F(s)G(s)$. Do this by writing $F(s) = \displaystyle\int_0^\infty f(x)e^{-sx}\,dx$ and $G(s) = \displaystyle\int_0^\infty g(y)e^{-sy}\,dy$; expressing the product as a double integral; and changing coordinates using $x = t - \tau$, $y = \tau$. The change of variables formula (as in Lecture 18 (Week 8) of the 2007 version of 18.02 on OCW, for example) will be very useful.

(c) Use the integral definition to find the Laplace transform of the function $f(t)$ with $f(t) = 1$ for $0 < t < 1$ and $f(t) = 0$ for $t > 0$. What is the region of convergence of the integral?

27. (M 12 Apr) [ODEs via Laplace transform] Let a and b be real numbers, with $a \neq 0$.

(a) Find the unit impulse response and unit step response for the first order operator $aD + bI$ by translating the problem to the frequency domain (i.e. apply the Laplace transform), solving there, and translating back to the time domain. (This will be much clearer if you use the rule $\mathcal{L}[\dot{x}] = sX(s)$ described in class and the Supplementary Notes, in which $\dot{x}$ is the generalized derivative of $x(t)$, than if you try to use the t-derivative rule as described in EP.)

(b) Solve $a\dot{x} + bx = t$ with rest initial conditions in three ways.
(i) Undetermined coefficients to get x_p, and add the appropriate transient.
(ii) Compute $w(t) * t$ (using the value for $w(t)$ you found in **(a)**).
(iii) Apply Laplace transform, solve, and transform back.

28. (W 14 Apr) [Second order ODEs via Laplace transform] Find the unit impulse response of the following operators by means of the Laplace transform.

(a) $3D^2 + 6D + 6I$.

(b) $D^4 + I$.

18.03 Differential Equations
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