

# Problem Set 2

1. Consider the following constrained optimization problem

$$P^* = \underset{-2 \leq x_1 \leq 2, -2 \leq x_2 \leq 2}{\text{minimize}} \quad (4 - 2.1x_1^2 + \frac{x_1^4}{3})x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2$$

1.a Reformulate the nonlinear optimization in terms of nonnegative polynomials.

1.b Reformulate the nonlinear optimization in terms of SOS polynomials.

1.c Solve the SOS program of the step 2.b (using e.g., Yalmip, SOSTools,...).

1.d Compare the obtained optimal objective function of the step 1.c with the optimal objective function of the step 2.a of Problem Set 1.

2. Consider the following nonlinear dynamical system

$$\dot{x}_1 = -x_1^3 - x_1 x_3^2$$

$$\dot{x}_2 = -x_2 - x_1^2 x_2$$

$$\dot{x}_3 = -x_3 - \frac{3x_3}{x_3^2 + 1} + 3x_1^2 x_3$$

2.a Write the valid SOS constraints for Lyapunov function  $V(x) = 5x_1^2 + 4x_2^2 + x_3^2$ .

2.b. Using the Lyapunov function  $V(x) = 5x_1^2 + 4x_2^2 + x_3^2$  show that the nonlinear system is stable.

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