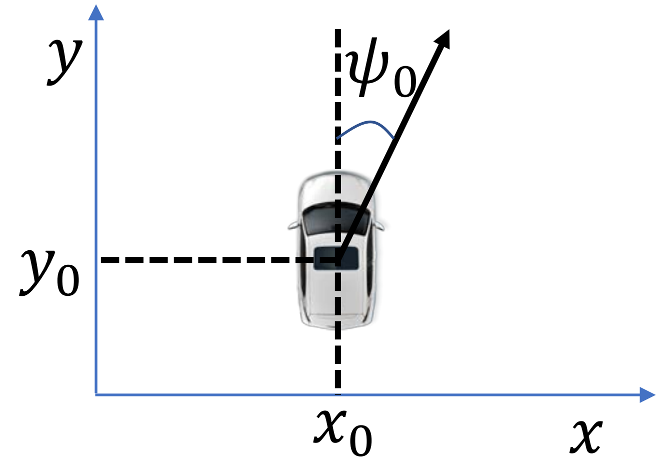
Problem Set 7

1. Consider the following uncertain nonlinear dynamical model of a vehicle:

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}, \qquad \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{w}) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -v\sin\psi + w \\ v\cos\psi \\ u \end{bmatrix}$$

$(x_0, y_0, \psi_0) = (0,0,0)$



- Control inputs u, v
- Source of Uncertainty: disturbance $\omega \in [-0.1 \;\; 0.1]$
- Obstacle: $X_{obs} = \{\; (x_1, x_2) \colon\; 0.25^2 - x_1^2 - (x_2 - 0.5)^2 \geq 0 \;\}$

- Motion Primitives :

1) $\psi_1^* = 0,\qquad v_1 = 1m/s, u_1 = -50(\psi - \psi_1^*),\qquad$ 2) $\psi_2^* = \dfrac{15\pi}{180},\qquad v_2 = 1.5m/s, u_2 = -50(\psi - \psi_2^*)$

3) $\psi_3^* = \dfrac{45\pi}{180},\qquad v_1 = 2m/s, u_3 = -50(\psi - \psi_3^*)\qquad$ 4) $\psi_4^* = \dfrac{90\pi}{180},\qquad v_4 = 3m/s, u_4 = -50(\psi - \psi_4^*)$

Check the safety of the given motion primitives.

Hint: Similar example on the page 33 of Lecture 8.

2. Consider the following uncertain nonlinear dynamical system

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = x_1(k)x_2(k) + u(k) + (0.2\omega(k) - 0.1)$$

- Source of uncertainties at time k: $(x_1(k), x_2(k), \omega(k)) \in \Omega_x = \{(x_1, x_2, \omega) = 0.1^2 - x_1^2 - x_2^2 - \omega^2 \geq 0\}$.
- Goal Set: Neighborhood of the way-point $(0,0.5)$, i.e. a ball around the way-point

$$X_{safe} = \{\, (x_1, x_2) \colon\ 0.2^2 - (x_1 - 0)^2 - (x_2 - 0.5)^2 \geq 0 \,\}$$

- Robust set for control input at time k :

$$U_R = \{u(k) : x(k+1) \in X_{safe}, \forall \omega \in \Omega_x\}$$

i) Find the inner approximation of the robust set of control input U_R using the SOS program with relaxation order $d = 4$.
ii) Obtain the Robust Set using Monte-Carlo Approach.

Hint: Similar example on the page 64 of Lecture 8.

3. Consider the following uncertain nonlinear dynamical system

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = x_1(k)x_2(k) + u(k) + (0.2\omega(k) - 0.1)$$

- Source of uncertainties at time k: $\big(x_1(k), x_2(k)\big) \sim U([-0.1, 0.1]^2)$ $\quad \omega_k \sim N(m, \sigma), m \in [-0.1, 0.1], \sigma \in [0.1, 0.3]$
- Unsafe set: $X_{obs} = \{ \ (x_1, x_2) \colon \ 0.3^2 - (x_1 - 0.2)^2 - (x_2 - 0.3)^2 \geq 0 \ \}$
- Distributionally Robust Chance constrained set for control input at time k:

$$U_{DR} = \{u(k) : \mathrm{Prob}(x(k+1) \in X_{safe}) \geq 1 - \Delta, \ \forall N(m, \sigma)\} \qquad \Delta = 0.2$$

i) Find the inner approximation of the set U_{DR} using the SOS program with relaxation order $d = 6$.

ii) Choose a control input $u_k \in U_{DR}$. Using the Monte-Carlo approach show that for the chosen controller the following result holds true:

$$Prob\big(x(k+1) \in X_{safe}\big) \geq 1 - \Delta$$

Hint: Similar example on the page 121 of Lecture 8.