Problem Set 6

1. Consider the following uncertain nonlinear dynamical system

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = x_1(k)x_2(k) + u(k) + (0.2\omega(k) - 0.1)$$

Where, (x_1, x_2) are the states, u is the control input, and ω is the probabilistic disturbance. Source of uncertainties are initial states $\big(x_1(0), x_2(0)\big)$ with known probability distribution $pr_{x_0}(x_1, x_2)$ and disturbance ω_k with known probability distribution $pr_{\omega_k}(\omega)$. Due to this uncertainties, states of the system at each time step k are also probabilistic.

We have a set of way-points to steer the sates of the system to the target point. Suppose that at time k states of the system have uniform probability distribution as $\big(x_1(k), x_2(k)\big) \sim U([-0.1, 0.1]^2)$ and disturbance $\omega_k \sim Beta(5,2)$. Also, way-point at time $k+1$ is given as $(0, 0.5)$.

We want to find the control input at time k, i.e., u_k, such that states $(x_1(k+1), x_2(k+1))$ reach the neighborhood of the way-point $(0, 0.5)$, i.e. a ball around the way-point $0.2^2 - (x_1 - 0)^2 - (x_2 - 0.5)^2 \geq 0$, with a high probability. For this, purpose:

i) Formulate this problem as a chance optimization and solve the moment SDP with the relaxation order of $d = 2$.
ii) Check the rank condition for the obtained moment vector.
iii) Approximate the control input with the first-order moment of the obtained moment vector.
iv) Compare the estimated probability of the success for the obtained control input using the Monte-Carlo approach.

Hint: Similar example on the page 94 of Lecture 7.

2. Consider the uncertain nonlinear system of problem 1. Unsafe set is defined as

$$X_{obs} = \{\ (x_1, x_2)\colon\ 0.3^2 - (x_1 - 0.2)^2 - (x_2 - 0.3)^2 \geq 0\ \}$$

Chance constrained set for control input at time k is defined as follows:

$$U_{cc} = \{u(k) : \text{Prob}(x(k+1) \in X_{obs}) \geq \Delta\}$$

Where, $\Delta = 0.1$

i) Find the outer approximation of the chance constrained set U_{cc} using the SOS program with relaxation order $d = 5$.

ii) Choose a control input as $u_k \notin U_{cc}$. Using the Monte-Carlo approach show that for the chosen controller the following results hold true:

$$u_k \notin\ U_{cc}:\ Prob(x(k+1) \in X_{obs}) \leq \Delta \quad \text{and} \quad Prob\big(x(k+1) \in X_{safe}\big) \geq 1 - \Delta$$

Hint: Similar example on the page 143 of Lecture 7.

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