

14.12 Midterm Exam #1  
October 8, 2009

**Answer all questions.** You have 85 minutes in which to complete the exam. Please show your calculations and provide rough explanations where you can't give formal statements so I can give you partial credit.

1. (15 Minutes – 20 Points) Answer each of the following subquestions *BRIEFLY*.

(a) (5 points) In my second lecture I defined a solution concept called “pure strategy iterated strict dominance.” In my fifth lecture I defined a more powerful version of iterated strict dominance. What was the difference between them and why was the game below a useful example?

	<i>L</i>	<i>C</i>	<i>R</i>
<i>U</i>	10, 9	10, 6	10, 10
<i>M</i>	-5, 9	15, 10	11, 12
<i>D</i>	-35, 10	10, 7	15, 5

(b) (5 points) Find all **pure strategy** Nash equilibria of the game below.

	<i>X</i>	<i>Y</i>	<i>Z</i>
<i>A</i>	4, 3	5, 2	5, 1
<i>B</i>	1, 7	6, 6	4, 3
<i>C</i>	2, 3	7, 3	5, 3

(c) (5 points) In class I discussed the general discrete choice model of price competition between  $N$  firms: the firms choose prices  $p_1, \dots, p_n$  and each consumer  $i$  decides to purchase from the firm  $j$  for which  $v - p_j + \epsilon_{ij}$  is largest. Describe briefly what happens to equilibrium prices as the number of firms  $N$  goes to infinity both for uniformly distributed  $\epsilon_{ij}$  and under general distributions.

(d) (5 points) Describe precisely an example of a game that has no pure or mixed strategy Nash equilibrium. Describe as well as you can a theorem that provides conditions under which a game with an infinite number of pure strategies must have a Nash equilibrium. What conditions of your theorem are violated in your example?

2. (25 Minutes – 30 Points)

When my daughter Anna was 3 years old, she liked to play Rock-Paper-Scissors. However, she faced a difficulty – three year olds find it hard to make “scissors” with their fingers.

Suppose that we capture this problem by treating her playing Rock-Paper-Scissors against her older sister using the asymmetric  $3 \times 3$  game shown below (with Anna as player 1).

	<i>R</i>	<i>P</i>	<i>S</i>
<i>R</i>	0, 0	-1, 1	1, -1
<i>P</i>	1, -1	0, 0	-1, 1
<i>S</i>	$-1 - c, 1$	$1 - c, -1$	$-c, 0$

(a) (13 Points) Consider first the version of this game where  $c > 1$ . (You can think of this as a model for the extreme situation where Anna is physically incapable of playing scissors.) Find a mixed-strategy Nash Equilibrium of this game.

(b) (3 points) What is Anna’s expected payoff in the equilibrium you found in part (a)?

(c) (14 points) Consider now the version of this game with  $0 < c < 1$ . Find a mixed strategy Nash equilibrium of this game in which both players play every strategy with positive probability.

3. (15 Minutes – 22 Points)

Two students are deciding how long to spend studying for 14.12 on the night before the exam. Let  $e_i$  be the fraction of the available time student  $i$  devotes to studying with  $0 \leq e_i \leq 1$ . Assume that the students' utilities are

$$\begin{aligned}u_1(e_1, e_2) &= \log(1 + 3e_1 - e_2) - e_1 \\u_2(e_1, e_2) &= \log(1 + 3e_2 - e_1) - e_2\end{aligned}$$

(A story for this would be that the first term reflects the benefits they get from learning and getting a good grade, whereas the second reflects the opportunity cost of time. The negative effect of  $e_2$  on student 1's utility could reflect that student 1 will get a lower grade if student 2 studies more and does better on the exam.)

- (a) (5 points) What is player 1's best response to a choice of  $e_2$  by player 2.
- (b) (13 points) Find a pure strategy Nash equilibrium of the game where players 1 and 2 choose  $e_1$  and  $e_2$  simultaneously.
- (c) (4 points) Is this game solvable by iterated strict dominance? How do you know this?

4. (25 Minutes – 28 Points)

Suppose Prof. Ellison decides to run a classroom experiment to teach about mixed strategy equilibrium (and make some money). He chooses two students from the class. Each student is required to write down an integer from 1 to 100 inclusive. The rules of the game are that the student who writes down the smaller number must pay Prof. Ellison that number of dollars. The student who writes down the larger number pays nothing. If both students write down the same number assume that both pay.

Assume that both students are risk-neutral and self-interested so that this game can be represented as  $S_1 = S_2 = \{1, 2, \dots, 100\}$  with

$$u_1(s_1, s_2) = \begin{cases} -s_1 & \text{if } s_1 \leq s_2 \\ 0 & \text{if } s_1 > s_2 \end{cases}$$

$$u_2(s_1, s_2) = \begin{cases} -s_2 & \text{if } s_2 \leq s_1 \\ 0 & \text{if } s_2 > s_1 \end{cases}$$

- (a) (3 points) Are any strategies in this game strictly dominated?
- (b) (7 points) This game has two pure strategy Nash equilibria. What are they?
- (c) (3 points) Discuss briefly why you should expect given what I told you in part (b) that this game would also have a mixed strategy Nash equilibrium.
- (d) (15 points) Find a symmetric mixed strategy Nash equilibrium of this game.

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