

Practice Problems for Insurance, Signaling

December 14, 2016

Part I

Short Questions: Answer 4 of 6 questions [5 points each, 20 points total]

1. True/False/Uncertain and why?: If people are worried about adverse selection in the health insurance market, then a plan of introducing more competition among many insurance providers (by, for example, forcing them to have similar plans and letting the buyers choose the cheapest) would likely solve the problem.

Solution: More competition will reduce the price but will not solve the problem of adverse selection. For the same plan, competition will make the price of the plan actuarially fair (so that none of the insurance companies make any profit). But whether individuals will choose a particular plan still depends upon their risk types. At any given price, individuals will buy an insurance plan if their expected benefit exceeds the price of the plan. Since individuals still know about their risks better than insurance companies, only higher risk individuals will buy a given plan. This increases cost of providing a plan as the average individual who purchase the plan has higher risk than average individual in the population. Since insurance companies need to break even in expectation, this will raise the actuarially fair price of the plan to the point where the average payoffs to those who enroll is same as the the price of the plan. Hence, perfect competition will not solve the problem of adverse selection.

2. A set of risk averse consumers, each with identical preferences and identical wealth $w > 0$, faces identical independent risks of experiencing loss $w > L > 0$. True, false, or uncertain and why: There are no potential gains from trade in risk among these consumers.

Solution: *False. They can pool risk to mutually insure*

3. True, false, uncertain and why: A central requirement for a valid regression discontinuity design is that the treatment and control groups are identical in every way but they are treated differently due to randomization of the ‘running’ variable (e.g., credit score, age, etc.).

Solution: *False. The treatment and control groups are not identical: they have very slightly different values of the running variable, and the running variable is not randomly assigned.*

4. The film studio Twentieth Century Fox once insured actress Betty Grables’ legs for \$1 million each (~\$17 million in current USD). Is this insurance arrangement likely an example of risk pooling, risk spreading, or risk transfer? Explain.

Solution: *The insurance policy is very unlikely to be an example of risk pooling; there were not a large number of other actresses whose legs would be insured (such that the insurer could pool the risk). This policy is very unlikely to be an example of risk spreading. The policy was sold on the private market; it was not a form of social insurance, involving transfers among citizens.*

The most plausible explanation for this policy is risk transfer. Twentieth Century Fox faced a small chance of experiencing a large financial loss if their prime asset (Grables’ million dollar legs) became unsuitable for movie-making. Twentieth Century Fox was able to buy insurance because there were gains from trade between the studio and the insurer. The studio’s maximum willingness to pay to avert the risk exceeded the insurer’s minimum willingness to accept payment to bear the risk. Thus, the insurer acted like a relatively risk-tolerant agent buying risk from a less risk-tolerant agent.

5. When hiring young lawyers, law firms attempt to screen out applicants who plan to have children during the first ten years of their career (call these applicants “family types”). This is because lawyers tend to reduce their billable hours and thus generate less profit for their law firms once they have children. True, false, uncertain or why: The *full disclosure* principle says that job applicants will explicitly or implicitly reveal their type (family type, non family type) during the interview process, and hence screening for this characteristic is not necessary.

Solution: *This would only be true if lawyers had a credible and costless way to reveal their types. If there is no credible way to reveal this information, or if credibly revealing it has a positive cost, the full disclosure principle is not guaranteed to hold.*

6. State Lotteries sold \$53 billion of lottery tickets in 2010. Suppose the cost of playing is \$1 and the expected value of playing is 50 cents (i.e. $\sum_i p_i W_i = \$0.50$ where W_i is payout i , and p_i is probability of that payout). Mr. Jo, a VNM expected utility maximizer, is indifferent between playing the lottery or not. Is Mr. Jo risk-averse, risk-loving, neither, or is there not enough information to say? Explain.

Solution: Mr. Jo's certainty equivalent of playing the lottery is \$1 (since he is indifferent between playing or not), so $CE[\text{Lottery}] > EV[\text{Lottery}]$ and the risk premium is -50 cents. This makes Mr. Jo a risk-lover (even though he may not actually play the lottery).

Part II

Body Cameras [30 points]

Suppose that Congress approves funds for 50,000 body cameras to be used by police officers across the U.S. There are 3,000 counties in the U.S. but only enough cameras initially for 1,000 counties. To maximize effectiveness, body cameras will only be used initially in the counties with the highest crime rates as measured by homicides per 10,000 persons. Specifically, the assignment rule is:

$$C_i = \begin{cases} 1 & \text{if } H_i \geq 0.65 \\ 0 & \text{if } H_i < 0.65 \end{cases}$$

where C_i denotes whether county i is allocated body cameras and H_i is the homicide rate per 10,000 persons in county i . There are exactly 1,000 counties with $H_i \geq 0.65$. You are a grad student at MIT (sorry) and you work for David Autor (sorry). President Obama has personally asked you to evaluate the program, finding its effects on incidents of police brutality.

1. Suppose that you have data on reports of police brutality (B_i) in each county i one year after the policy was introduced. Explain carefully how you would construct a regression discontinuity estimator of the effect of the cameras on incidents of police brutality. Call this estimator T_{RD} . **(5 points)**

[Solution: intuitively, the estimator comes from comparing incidents of brutality in counties just above the homicide threshold to those just below it. Formally our estimate is

$$T_{RD} = \lim_{\epsilon \downarrow 0} E[B_i | H_i = 0.65 + \epsilon] - \lim_{\epsilon \uparrow 0} E[B_i | H_i = 0.65 + \epsilon]$$

which in the limit is equal to

$$T = E [B_{i1} - B_{i0} \mid H_i = 0.65]$$

with potential outcomes notation such that B_{i1} is police brutality in county i if this county received police cameras.]

2. Does T_{RD} provide an unbiased estimate of the causal effect of receiving police cameras on police brutality? Explain why or why not, along with any assumptions you need to make. **(6 points)**

[Solution: yes, we have an unbiased estimate provided that potential outcomes are continuous around the homicide rate threshold, i.e. that

$$\lim_{\epsilon \downarrow 0} E [B_{i1} \mid H_i = 0.65 + \epsilon] = \lim_{\epsilon \uparrow 0} E [B_{i1} \mid H_i = 0.65 + \epsilon]$$

$$\lim_{\epsilon \downarrow 0} E [B_{i0} \mid H_i = 0.65 + \epsilon] = \lim_{\epsilon \uparrow 0} E [B_{i0} \mid H_i = 0.65 + \epsilon]$$

For this, we would like that no other policies that may affect incidents of police brutality vary discontinuously around the 0.65 threshold (so we should check this).]

3. Suppose that you find $T_{RD} = -5$. President Obama thinks that this is strong enough evidence to support the expansion of the policy to the remaining counties in the U.S. Do you think that T_{RD} is an unbiased estimate of the causal effect for these remaining counties with $H_i < 0.65$? Why or why not? **(6 points)**

[Solution: T_{RD} gives us an unbiased local causal effect – that is to say, it is unbiased for counties with $H_i = 0.65$. Extrapolating away from the cutoff requires more assumptions, so we cannot say much about what the policy’s effects will be for counties that start off with lower homicide rates. For one thing, we might think that the number of police brutality incidents tends to be lower in counties with the lowest homicide rates. It follows that the treatment effect may be smaller in these places. In particular, if the initial number of incidents in some set of counties is less than 5, clearly the treatment effect of cameras cannot be -5 here since the number of incidents is bounded below by zero.]

4. You discover another policy that varies at the threshold: counties with $H_i \geq 0.65$ are allowed to use armored police cars, whereas counties with $H_i < 0.65$ are not allowed. This policy was introduced two years before the body cameras. When will this be a problem for your estimates? Can you suggest a test to check for this problem? [Hint:

you can assume that you have data on police brutality both for the years before and the years after the cameras were introduced.] **(6 points)**

[Solution: this will be a problem if it leads to a violation of potential outcomes being equal on either side of the threshold. This will be the case if armored police cars also have an effect on the number of incidents of police brutality (e.g. an increase because the use of ‘unnecessary’ armored police cars reduces the respect people have for the police). Since the policy was introduced two years before the body cameras were, we can calculate a regression discontinuity estimate for one of these two ‘pre-years’. Specifically we would estimate

$$T_{RD} = \lim_{\epsilon \downarrow 0} E[B_{iy} | H_i = 0.65 + \epsilon] - \lim_{\epsilon \uparrow 0} E[B_{iy} | H_i = 0.65 + \epsilon]$$

where y is one of the two years prior to the introduction of body cameras, but after introduction of armored cars.]

5. Suppose that you find that cameras reduce the number of police arrest incidents that result in death. Some years later, many (but not all) other counties decide independently to spend the money to buy cameras. How might you use this information to estimate the value of statistical life? Would your estimate be a lower or upper bound, or neither? Explain carefully the assumptions that would need to hold for your estimate to be valid. **(7 points)**

*[Solution: this is similar in flavor to Ashenfelter and Greenstone (2004) – in their case, some states show willingness to raise speed limits, accepting greater risk of death in exchange for a certain amount of time savings. In this case we have the opposite – some counties show willingness to spend money in exchange for a lower risk of death. Suppose the cost per year of police cameras is C and the RD estimate for the reduction in annual arrest incidents that result in death is R . In this case we have a **lower** bound for the VSL as $\frac{C}{R}$. For this to be valid, we need the following assumptions:*

- *Counties make voluntary decisions (not under pressure from Federal government for example)*
- *Our RD estimate for R is unbiased not only for the local effect, but also for the average effect for the untreated counties that decide to buy police cameras*
- *Counties make informed decisions – knowing R and C (the effect of cameras on incidents that result in death) in advance*
- *Counties consider the only benefits of the policy to be the reduction in deaths (there are no additional political benefits for example)*

Alternatively, we could discard the RD estimate of R and instead calculate a diff-in-diff estimate of R (as in Ashenfelter-Greenstone) using the fact that not all of the ‘control’ counties buy the body cameras. We may think that this is a more credible estimate of the effect on deaths for these counties, given that the RD estimate is really a local estimate (as discussed above).]

Short Questions (Choose any 4 of 6: 5 points each)

Please answer True/False/Uncertain and provide a brief explanation. *Correct answers without explanation will not receive credit.*

1. Ashok and Sahar are risk-averse Von Neumann Morgenstern expected utility maximizers who have utility of wealth $U(w) = \sqrt{w}$. Each faces a 50% probability of having wealth of \$1,600 tomorrow and a 50% probability of having wealth of \$900 tomorrow. The probabilities that Ashok and Sahar receive \$1,600 and \$900 are independent. Evaluate the following statement: Ashok and Sahar could unambiguously increase their expected utility by agreeing today to split whatever wealth they have tomorrow. You may explain your answer graphically, mathematically, or through careful verbal exposition. [Note: If you choose to answer mathematically, it may be helpful to know that $\sqrt{1250} \approx 35.35$]

True. Note that $U(1,600) = 40$, $U(900) = 30$, and $U(0.5 \times (1600 + 900)) = 50 \times \sqrt{1/2} \simeq 35.35$. Their expected utilities when pooling their incomes are:

$$U(L_p) = 0.5^2 \times 40 + 0.5^2 \times 30 + 2 \times .5^2 \times 35.35 = 35.17$$

Their expected utilities when not pooling are:

$$U(L_0) = .5 \times 40 + 0.5 \times 30 = 35$$

*Although pooling **does not eliminate risk** for either party, it reduces the probabilities of extreme outcomes, increases the probability of intermediate outcomes, and has no effect on the average (expected) outcome. Thus, it unambiguously reduces risk for both parties.*

- (a) U.S. military veterans who are diagnosed with diabetes receive comprehensive, free Veterans Healthcare (similar to Medicare for Veterans). To obtain a diagnosis of diabetes, a veteran must have a blood glucose level of at least 126 ml/DL. It is common knowledge among veterans that drinking several cans of Coke (*not* Diet

Coke) the night before the test will elevate their blood glucose levels, increasing the odds that they receive a diabetes diagnosis. A researcher suggests using Veterans' glucose levels as a Regression Discontinuity design for studying the effect of Veterans Healthcare on health. Specifically, she proposes comparing the health of Veterans with tested blood glucose levels *just below* 126 to those with tested blood glucose levels *just above* 126 to isolate the causal effect of receiving Veterans Healthcare on health. Evaluate the following statement: This design is likely to provide an unbiased estimate of the causal effect of interest.

False. Assignment to treatment status is endogenous. Veterans who are very motivated to obtain Veterans Healthcare can manipulate their blood glucose scores by pounding down Coke the night before the test to increase their odds of a diabetes diagnosis. It's likely that less healthy veterans will be disproportionately likely to choose to "beat the test." This means that those who just qualify for Veterans healthcare are likely not comparable to those who are just below the qualification level. Intuitively, the ability of Veterans to manipulate their own test scores means that Veterans Healthcare is not effectively randomly assigned for Veterans who are arbitrarily close to the 126ml/DL threshold.

- (b) Coal miners earn \$100,000 per year and face an annual risk of fatal on-the-job injury of 24.8 deaths per 100,000. Managers of coal mines earn \$100,000 per year and face an annual risk of fatal on-the-job injury of 4.3 deaths per 100,000. Evaluate the following statement: We can infer that the Value of a Statistical Life for coal miners is approximately zero.

False. Unlike the Ashenfelter/Greenstone speed limit example, we do not observe any individual (or U.S. state) making an explicit trade-off between income (or time) and safety; it's possible—in fact, likely—that coal miners would earn less than \$100K in a safer job and managers of coal mines would earn more than \$100K in a riskier job. Thus, the simple correlation between salary and risk (which is in this case zero) does not provide any meaningful information about the willingness of individuals to pay at the margin for safety. Consider, for example, that the average CEO earns at least a \$1 million per year and faces less than a 1 in 100,000 annual risk of fatal on-the-job injury. Following the (erroneous) logic of the question, by comparing managers of coal mines to CEOs, we might conclude that the VSL for managers of coal mines is negative.

II.1 Insurance markets (25 points)

MIT offers a basic dental insurance plan for graduate students. Suppose the only dental procedure that this insurance covers is a tooth filling. A filling costs M dollars. Graduate students have different probabilities of needing a filling. Denote the probability that student i needs a filling as θ_i . This probability is distributed uniformly on the unit interval, i.e. $\theta \sim U[0, 1]$. Assume all graduate students have initial wealth w . All graduate students are von Neumann Morgenstern expected utility maximizers with utility of wealth equal to $\ln(w)$. Throughout the problem, assume that students know their risk type θ .

1. Suppose MIT didn't offer dental insurance. What would be the expected utility of a student that has the average risk ($\theta_i = 0.5$) of needing a filling?

For a student with the average risk type, the probability of needing a filling is $E[\theta] = 0.5$. Thus, her expected utility is $EU = 0.5\ln(w) + 0.5\ln(w - M)$

2. Suppose that MIT knew the probability that each student i needed a filling and offered dental insurance to each student at an actuarially fair premium π_i (note that the premium π_i depends on θ_i). How much insurance I would a student of risk type θ_i buy? [Hint: $\pi_i = \theta_i I$]. Provide the algebraic derivation of your result. In general, would you need to do any algebra to arrive at your conclusion?

The expected utility of risk type θ with insurance is: $EU = (1 - \theta)\ln(w - \theta I) + \theta\ln(w - \theta I - M + I)$. The student will maximize this expression subject to I in order to find the optimal insurance amount:

$$\max (1 - \theta)\ln(w - \theta I) + \theta\ln(w - \theta I - M + I)$$

FOC:

$$\frac{-(1 - \theta)\theta}{w - \theta I} + \frac{\theta(1 - \theta)}{w - \theta I - M + I} = 0$$

$$\frac{1}{w - \theta I - M + I} = \frac{1}{w - \theta I}$$

$$w - \theta I = w - \theta I - M + I$$

$$I = M$$

Thus, the risk-averse students of any type would want to buy full insurance at the actuarially fair price. We didn't have to do the algebra here, since we know the general result that risk-averse individuals want to buy full insurance at an actuarially fair price.

3. Suppose that MIT only knew the distribution of probabilities that students needed

fillings, but did not know each student's individual risk. Suppose MIT charged a premium of $\pi = 0.5M$, equal to the expected cost of the average student. Thus, this policy costs $0.5M$ and pays a benefit of M if the student needs a filling. Lastly suppose that $w = 2$ and $M = 1$. Which graduate students (that is, what risk types) would buy this insurance policy? [You may make use of the fact that $\frac{\ln(1.5)}{\ln(2)} \approx 0.6$] Would MIT break even, make positive profits, or lose money on this dental policy?

Only those students that have higher expected utility with insurance at the offered premium than expected utility without insurance will buy insurance. The student's expected utility without insurance is: $EU_{NI} = (1 - \theta)\ln(w) + \theta\ln(w - M)$. The student's expected utility with full insurance insurance at the offered premium is $EU_I = (1 - \theta)\ln(w - 0.5M) + \theta\ln(w - 0.5M - M + M) = \ln(w - 0.5M)$. Thus, a student of risk type θ will buy insurance if:

$$\ln(w - 0.5M) > (1 - \theta)\ln(w) + \theta\ln(w - M)$$

$$\ln(2 - 0.5) > (1 - \theta)\ln(2) + \theta\ln(2 - 1)$$

$$\ln\left(\frac{3}{2}\right) > (1 - \theta)\ln(2)$$

$$\theta > 1 - \frac{\ln(1.5)}{\ln(2)} \approx 0.4$$

The average risk of MIT's recruited risk pool doesn't have the same expected cost as what the actuarially fair premium for the whole population of students is set to cover; thus, this plan would lose money in expectation.

- Now suppose that MIT can observe for each student i whether the student's risk type θ_i is above or below 0.5. MIT cannot, however, observe the actual value of θ_i for any student. Let us call students with $\theta_i \geq 0.5$ H -types (for high risk) and students with $\theta_i < 0.5$ L -types (for low risk). MIT decides to offer two dental insurance plans, one each for H -types and for L -types. It charges premiums $\pi_L = 0.25M = 0.25$ to Low types and $\pi_H = 0.75M = 0.75$ to High types. MIT only sells the H policy to H -types and only sells the L policy to L -types (remember that MIT can tell them apart). Draw a diagram with the uniform distribution of types that shows which risk types will buy insurance and which risk types will not. Your diagram *must* mark explicitly where the types that buy insurance are located on the distribution relative to the average risk type ($\theta_i = 0.50$) and relative to the risk types $\theta_i = 0.25$ and $\theta_i = 0.75$. In words, explain which individuals will now be buying insurance and compare them to the set of individuals that were buying insurance in part (3). You do not need to do any algebra

here, but you must be quite precise in your diagram and in your verbal explanation.

We would expect the students to indicate that the marginal enrollee of each risk type is going to be located to the left of the actuarially fair premium for their respective groups due to the assumed risk aversion.

5. In your diagram in part (4), you will notice that students can be classified into several groups according to (1) their risk type (H or L) and (2) whether or not they buy insurance. Define (name or label) these groups. For each group, state whether the members of the group are unambiguously worse off, unambiguously better off, or if you cannot tell, under the two-policy scheme than under the one-policy scheme. You must provide an explanation for your conclusions. (You don't need to do any algebra)

We have four distinct groups under the two-policy scheme: high types that buy insurance, high types that don't buy insurance, low types that buy insurance, low types that don't buy insurance. Under the two-plans scheme, as compared to one actuarially-fair plan, more individuals in the lower risk category get insurance, but some individuals with higher risk don't get insurance. Further, low risk individuals that do get insurance under the two-plan scenario pay less for their insurance, while the highest risks pay more. Low risk types are better off here - those that were insured before are now paying lower premium and some low types that were not insured before gained access to insurance. High types are worse off - some of them don't have insurance anymore and the ones that do pay a higher premium for it. Low risk types are better off, since they are not pooled with high types and thus don't have to cross-subsidize them. High types are now worse off, since they are not pooled with low risk types and thus don't get the cross-subsidy, having to pay more for their coverage. Note that low types get to enjoy their cheap coverage of full insurance because we assumed that MIT can impose that High types are not allowed to buy the policy intended for the Low types.

II.2 Banking at night (20 points)

Investment banks are interviewing applicants for analyst jobs that will require them to work very long hours. Applicants differ only according to their ability to work long hours. Denote this ability with η and suppose it is uniformly distributed on $[0, 1]$. Applicants get utility w_h if hired by a bank at wage w_h (measured in dollars). They get utility of zero if they are not hired. If a bank hires someone with ability η , she produces output $8\eta - 6$ (measured in dollars). Applicants know their own type η , but banks cannot tell an applicant's type during an interview. All applicants and all banks know the distribution of η . They also know that if the job interview is held at 1am (that is, late at night), it costs $(1 - \eta)$ for an applicant of type η to attend the interview.

1. If banks only hired applicants who attended the 1am interview and paid them a wage of w_h , which applicants would apply for the job?

A student of type η knows that she will be hired only if she attends the interview. If the student doesn't attend the 1am interview, she is not hired and receives utility zero. If she attends the interview, she receives utility $w_h - (1 - \eta)$. A student will attend the interview if and only if $w_h - (1 - \eta) > 0$, equivalently $\eta > 1 - w_h$.

2. What is the equilibrium wage w_h that banks will offer to applicants who come to the interview? To solve this problem, assume that: banks have also solved part (1) above, and that due to competition among banks for applicants, banks will make zero expected profit in equilibrium.

By offering a wage of w_h , the bank obtains output of:

$$Y(w_h) = 8E(\eta|w_h) - 6$$

$$Y(w_h) = 8E((1 + 1 - w_h)/2) - 6$$

$$Y(w_h) = 8 - 4w_h - 6$$

$$Y(w_h) = 2 - 4w_h.$$

The zero profit condition means that $Y(w_h) = w_h$. Hence, $2 - 4w_h = w_h$, so $w_h = 2/5$. To check that this is an equilibrium, first note that the average quality of applicants at wage w_h is $E[\eta|\eta > 1 - 2/5] = ((1 + 3/5)/2) = 4/5$. Average output is $32/5 - 6 = 2/5 = w_h$. Thus, the zero profit condition holds. An applicant with $\eta < 3/5$ would not apply for the job since that worker's cost would exceed the wage of $2/5$. Applicants who have $\eta \geq 3/5$ at least weakly prefer to interview for the job. Note also that an applicant with

$\eta = 3/5$ is indifferent between applying for the job and not applying for the job. Thus, this is a separating equilibrium.

3. Now suppose that instead of conducting interviews, the bank offers to hire any applicant that wants a job and promises to pay a wage that is equal to the worker's individual output when the work is complete. Assume that once the new hire starts working, the bank can directly measure her output. Who will apply for the job in this case?

Individuals who will be paid non-negative wages will want to take the bank's offer. Since wages are determined by actual productivity, only individuals with non-negative output will want to participate. This is equivalent to the following condition:

$$\begin{array}{r} 8\eta - 6 \geq 0 \\ \eta \geq \frac{3}{4} \end{array}$$

4. Notice that the group of individuals that are hired through the interview process are not identical to those hired through the process suggested in part (3). In particular, the set of workers hired in part (3) will have higher average productivity than those hired in part (2). Provide intuition for why this occurs.

The interview recruitment does not reveal actual productivity, so the firm must still pay workers according to their expected productivity, given the group of workers who is employed. Workers will show up to the interview as long as the cost of doing so does not exceed the wage. This will induce some workers who actually produce negative output to show up to the interview. In the internship case, workers reveal their true productivity through the internship and then are paid according to their true output. Thus, workers will not want to participate in the internship unless they will be paid a non-negative wage. This would screen out workers who produce negative output. In both cases workers are paid according to their expected productivity, but in the internship case, an information asymmetry no longer exists, so the firm can pay workers according to their true productivity.

III. Causal inference (20 points)

Under the No Child Left Behind legislation, students in grades 3 through 8 are required to take standardized tests on an annual basis. Every student receives a test score and a performance label, which is assigned entirely based on test score cutoffs. There are two labels that a student may receive: “proficient” and “failing”. The only benefit of scoring above a cutoff is receiving a more positive performance label. In this problem, we will explore the extent to which a more positive performance label affects students’ test scores in subsequent years.

For simplicity, let’s focus on students’ performance labels in the seventh grade only. We will denote a student’s seventh grade test score by D_i , where $D_i \in [0, 100]$. If a student’s test score is 50 or above, she receives a “proficient” performance label, and if her test score is 49 or below, she receives a “failing” performance label. We can represent this succinctly as:

$$P_i = \begin{cases} 1 & \text{if } D_i \geq 50 \\ 0 & \text{if } D_i < 50 \end{cases}$$

Let Y_i denote a student’s eighth grade test score, where $Y_i \in [0, 100]$.

Suppose initially that students are notified of their seventh grade test scores but not their performance labels (i.e. there is no student behavioral change associated with a more/less positive performance label). Further suppose that students’ eighth grade teachers do *not* know students’ test scores, but *do* know their performance labels. The mechanism we are attempting to isolate is whether students’ eighth grade teachers treat students differently based on their performance labels from the seventh grade.

1. Consider the following approach for estimating the effect of seventh grade performance labels on students’ eighth grade performance. We compare eighth grade test scores of students with $P_i = 1$ to students with $P_i = 0$. Write this estimator, \hat{T} , in formal notation. Does \hat{T} provide an unbiased estimate of the causal effect of receiving a more positive performance label on students’ subsequent test scores? Why or why not?

$\hat{T} = E[Y|P = 1] - E[Y|P = 0]$. *\hat{T} does not provide an unbiased estimate of the causal effect of performance labels. Students who are labeled as proficient likely possess higher innate ability levels, on average, than students who are labeled as failing. Given that students who are labeled as proficient are likely of higher ability, their eighth grade test scores will likely be higher on average than the test scores of students who were*

labeled as failing. If that is the case, even if performance labels had no causal effect on eighth grade test scores, this would result in a positive estimate of \hat{T} . Invoking formal notation, our causal effect of interest is:

$$T = E[Y_1|P = 1] - E[Y_0|P = 1]$$

But we are estimating:

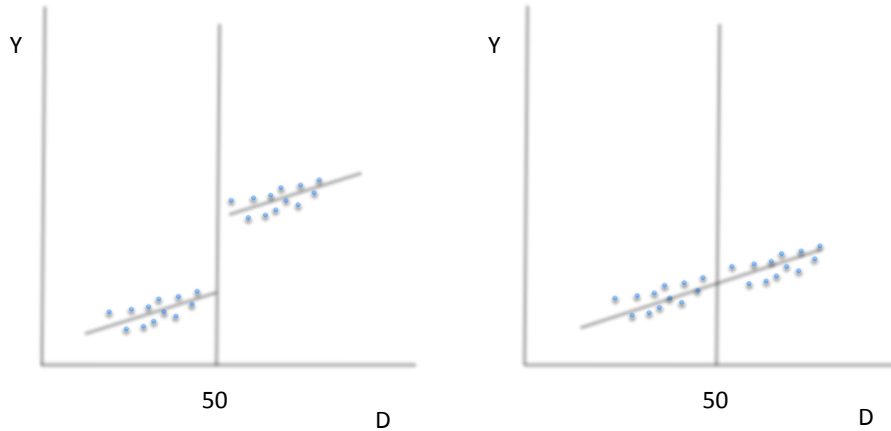
$$\hat{T} = E[Y_1|P = 1] - E[Y_0|P = 0]$$

This results in the following bias:

$$\begin{aligned} \text{Bias} &= \hat{T} - T \\ &= E[Y_0|P = 1] - E[Y_0|P = 0] \\ &\neq 0 \end{aligned}$$

In words: in the absence of performance labels, students who were labeled as proficient would have had higher eighth grade test scores than students who were labeled as failing. Thus, students who were labeled as failing do not provide a valid counterfactual for students who were labeled as proficient.

2. The Department of Education is concerned that performance labels will cause teachers to devote a disproportionate amount of their time to students labeled as “proficient.” Construct a graph, with D_i on the x axis and Y_i on the y axis, where D_i is student i 's seventh grade test score and Y_i is student i 's eighth grade test score. Plot a potential relationship between D_i and Y_i that would substantiate the Department of Education's concern. On another graph, plot a potential relationship between D_i and Y_i that would suggest that the Department of Education need not be concerned that teachers are devoting a disproportionate amount of time students labeled as “proficient.”



3. Explain how you would construct a regression discontinuity estimator of the effect of receiving a more positive performance label in the seventh grade on students' eighth grade test scores. Call this estimator T_{RD} . Does T_{RD} provide an unbiased estimate of the causal effect of receiving a more positive performance label on students' subsequent test scores? Why or why not?

Intuitively, we can construct an RD estimator by comparing the students who fall very close to either side of the threshold for the proficient label. Formally,

$$T_{RD} = \lim_{\epsilon \downarrow 0} E[Y|D = 50 + \epsilon] - \lim_{\epsilon \uparrow 0} E[Y|D = 50 + \epsilon]$$

T_{RD} will provide an unbiased estimate of the causal effect of performance labels on students' subsequent test scores as long as students who are very close to the threshold have the same potential outcomes:

$$\lim_{\epsilon \downarrow 0} E[Y_1|D = 50 + \epsilon] = \lim_{\epsilon \uparrow 0} E[Y_1|D = 50 + \epsilon]$$

$$\lim_{\epsilon \downarrow 0} E[Y_0|D = 50 + \epsilon] = \lim_{\epsilon \uparrow 0} E[Y_0|D = 50 + \epsilon]$$

The eighth grade test scores of students who barely missed the cutoff for the proficient label are compared to the eighth grade test scores of students who barely made the cutoff. The potential outcomes of these groups are arguably very similar, resulting in unbiased estimate of the causal effect of performance labels on subsequent test scores.

4. Suppose that due to a Department of Education oversight, half of the students who were very close to the “proficient” threshold were accidentally classified as “failing”. In this case, do you think that T_{RD} overstates or understates the effect of receiving a more positive performance label on subsequent test scores (or is it indeterminate)? Explain. *If the true causal effect of performance label is positive (due to, for example, teachers devoting more time to high-performing students), then this misclassification will result in an understatement of the true effect of performance labels on subsequent test scores. T_{RD} still compares students who are on either side of the 50 threshold, but half of the students who are above the threshold did not actually receive the treatment (performance labels). Note that if the true causal effect is negative, then this misclassification will still result in an understatement of the true effect (it will be less negative/biased toward zero).*

MIT OpenCourseWare
<https://ocw.mit.edu>

14.03 / 14.003 Microeconomic Theory and Public Policy
Fall 2016

For information about citing these materials or our Terms of Use, visit: <https://ocw.mit.edu/terms>.