

14.03/14.003 Recitation 5: General Equilibrium Practice Problem

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Consider an economy with two goods - X and Y - and two agents - Ann and Bob. Ann and Bob wish to trade with one another in order to maximize their individual utilities. We will consider how their trading decisions depend on the initial endowments of X and Y and on their utility functions.

Suppose Ann is endowed with one unit of X and half a unit of Y i.e. $e_{Ann} = (1, \frac{1}{2})$ and Bob is endowed with 1 unit of X and 1.5 units of Y i.e. $e_{Bob} = (1, \frac{3}{2})$. Additionally, suppose their utility functions are given by:

$$U_{Ann}(X, Y) = XY$$

$$U_{Bob}(X, Y) = Y + 2X$$

- 1 Draw an Edgeworth box indicating the endowment and preferences of this problem.

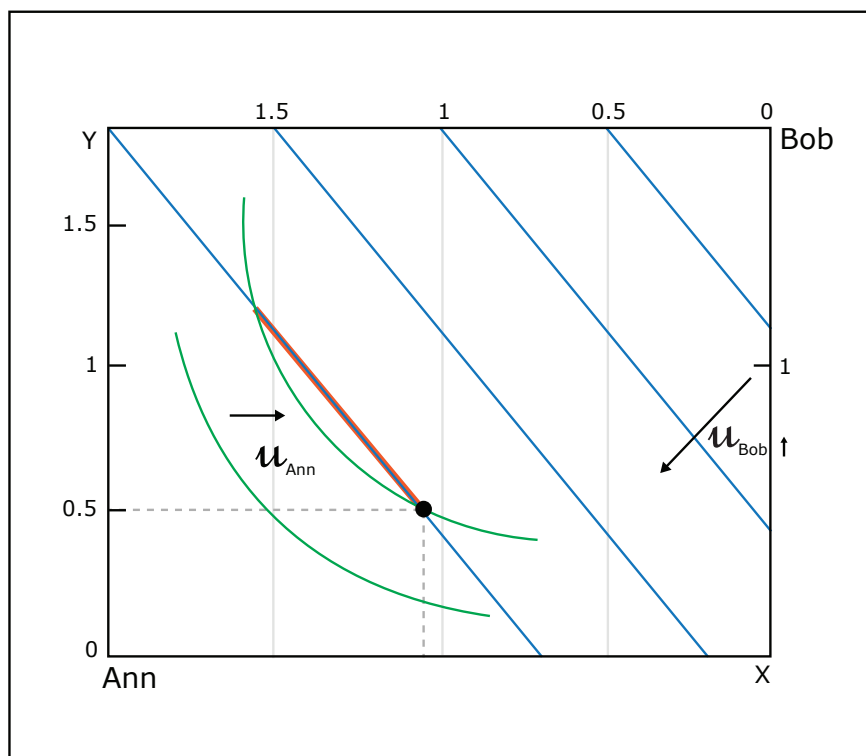


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2 Find the set of Pareto Optimal Allocations in this economy and depict these in the Edgeworth box. What is this set of points called?

- The contract curve is the set of points such that by moving away from these allocations, at least one of the individuals are made worse off.
- Visually, this is given by the set of tangencies of the consumer utility functions. The these tangencies, the consumers have the same MRS between X and Y .
- The MRS is defined as $\frac{\partial U/\partial X}{\partial U/\partial Y}$. For Ann, this is $\frac{Y}{X}$. For Bob, this is 2. Therefore, the set of all points where the MRS are equal are given by

$$\frac{Y_{Ann}}{X_{Ann}} = 2 \rightarrow Y_{Ann} = 2X_{Ann}$$

- Note, however, that in this economy, there are only 2 units of X and 2 units of Y , therefore, this equation cannot hold when $X_{Ann} > 1$. When $X_{Ann} > 1$, the contract curve is just going to be on the top edge of the box.

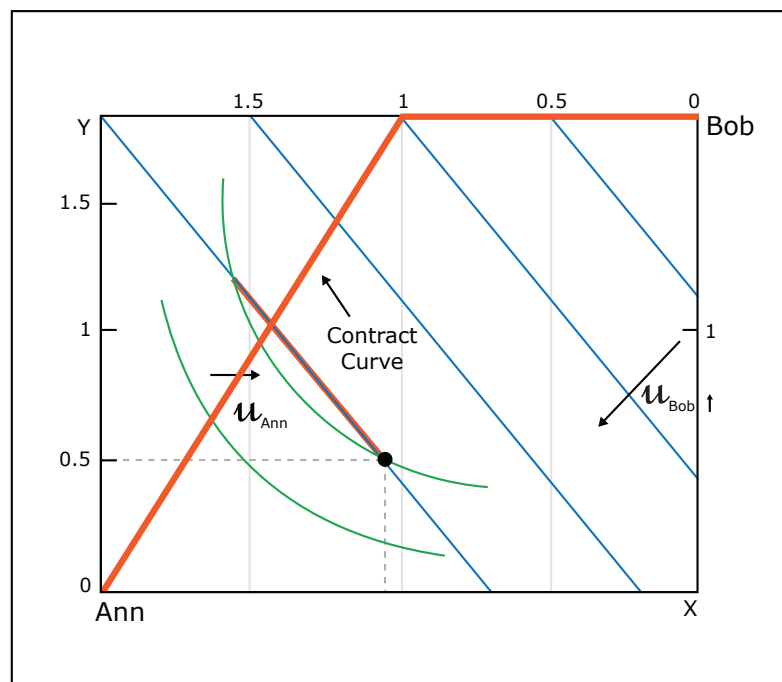


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3 Find the equilibrium consumption of X and Y by Ann and Bob in this economy and determine the price ratio that supports this equilibrium

- The competitive equilibrium allocation in this economy will satisfy 2 conditions:
 1. Both Ann and Bob are maximizing their utility subject to their budget constraint
 2. The final allocations of X and Y satisfy the resource constraint (i.e. they do not add up to more than 2)

- We begin with the first condition, that the individuals maximize their utility subject to their budget constraint. Note that in this case, their budget constraint is given not by their exogenously determined income I but rather by the combination of their endowment and the prices. In other words, their income is determined by the amount they would have if they sold their entire endowment at the given prices. Note that this means that their income changes as the price changes. The Lagrangian for Ann's problem is therefore given by:

$$L = X_{Ann}Y_{Ann} + \lambda(P_x E_X^{Ann} + P_Y E_Y^{Ann} - P_x X_{Ann} - P_y Y_{Ann})$$

$$L = X_{Ann}Y_{Ann} + \lambda(P_x + P_Y \frac{1}{2} - P_x X_{Ann} - P_y Y_{Ann})$$

- All that matters in this setting is the relative price (NOT the price level). Therefore, it is going to be very helpful to normalize one of the prices to 1. Let's choose $P_X = 1$. The solution to this Lagrangian will yield Ann's marshallian demands for good X and Y .

$$\frac{\partial L}{\partial X_{Ann}} = Y_{Ann} - \lambda = 0$$

$$\frac{\partial L}{\partial Y_{Ann}} = X_{Ann} - \lambda P_Y = 0$$

$$\frac{\partial L}{\partial \lambda} = 1 + P_Y \frac{1}{2} - X_{Ann} - P_Y Y_{Ann} = 0$$

- Solving these three equations for Y_{Ann} and X_{Ann} , we get:

$$X_{Ann} = \frac{2 + P_Y}{4}$$

$$Y_{Ann} = \frac{2 + P_Y}{4P_Y}$$

- Similarly for Bob, we have

$$L = Y_{Bob} + 2X_{Bob} + \lambda(P_x E_X^{Bob} + P_Y E_Y^{Bob} - P_x X_{Bob} - P_Y Y_{Bob})$$

or

$$L = Y_{Bob} + 2X_{Bob} + \lambda(1 + P_Y \frac{3}{2} - X_{Bob} - P_Y Y_{Bob})$$

$$\frac{\partial L}{\partial X_{Bob}} = 2 - \lambda = 0$$

$$\frac{\partial L}{\partial Y_{Bob}} = 1 - \lambda P_Y = 0$$

$$\frac{\partial L}{\partial \lambda} = 1 + P_Y \frac{3}{2} - X_{Bob} - P_Y Y_{Bob} = 0$$

Solving these, we get:

$$P_Y = \frac{1}{2}$$

and

$$X_{Bob} + P_Y Y_{Bob} = 1 + \frac{3}{2} P_Y$$

- Lastly, we use the second equilibrium condition (i.e. the resource constraints) to pin down the consumption bundles and the relative prices.

$$X_{Ann} + X_{Bob} = 2$$

$$Y_{Ann} + Y_{Bob} = 2$$

- Plugging all this in, we finally get:

$$\frac{P_Y}{P_X} = \frac{1}{2}$$

$$X_{Ann} = \frac{5}{8}, Y_{Ann} = \frac{10}{8}$$

$$X_{Bob} = \frac{11}{8}, Y_{Bob} = \frac{6}{8}$$

- Note, that P_Y (which is the relative price of good Y and X) clears both the market for X and the market for Y. This result is called Walras' law, which stipulates that if n-1 markets clear, the other market will definitely clear as well.

4 *Will this equilibrium allocation be Pareto Efficient?*

- Yes, and we can show this two ways. First, we can show that it is on the contract curve. Note that we found in part 2 that the contract curve is defined by

$$Y_{Ann} = 2X_{Ann}$$

and $\frac{5}{8} * 2 = \frac{10}{8}$. We can also show this without any math using the first welfare theorem. The first welfare theorem states that as long as certain conditions are satisfied, a competitive equilibrium is pareto efficient. The conditions are:

- No transaction costs
- No market power
- No externalities
- Full information
- Since the problem assumes that these assumptions are satisfied in our setting, we can conclude that the competitive equilibrium allocation is pareto efficient.

5 *Show that there are gains from trade in this setting. From what endowment point would there not be gains from trade?*

- There are gains from trade when the individuals are able to achieve higher utility levels after the trade than before the trade. We can easily see this in the figure, and we can also calculate the utility for each consumer at their endowment point and their equilibrium point.

$$U_{Ann}(e_X, e_Y) = \frac{1}{2}, U_{Ann}(X_{Ann}, Y_{Ann}) = \frac{50}{48}$$

$$U_{Bob}(e_X, e_Y) = \frac{7}{2}, U_{Bob}(X_{Bob}, Y_{Bob}) = \frac{28}{8} = \frac{7}{2}$$

Note that in this scenario, Ann benefits from the trade and Bob is indifferent. This is because of the shape of his indifference curve!

6 Suppose that instead of the preferences above, $U_B(X, Y) = X^2Y^2$. How does this change the equilibrium consumption that you found in part 3? What is the intuition for this?

- In this case, Bob's Lagrangian becomes

$$L = 2\log(X_{Bob}) + 2\log(Y_{Bob}) + \lambda(1 + P_Y \frac{3}{2} - X_{Bob} - P_Y Y_{Bob})$$

$$\frac{\partial L}{\partial X_{Bob}} = \frac{2}{X_{Bob}} - \lambda = 0$$

$$\frac{\partial L}{\partial Y_{Bob}} = \frac{2}{Y_{Bob}} - \lambda P_Y = 0$$

$$\frac{\partial L}{\partial \lambda} = 1 + P_Y \frac{3}{2} - X_{Bob} - P_Y Y_{Bob} = 0$$

This yields the following Marshallian demands:

$$Y_{Bob} = \frac{1 + \frac{3}{2}P_Y}{2P_Y}$$

$$X_{Bob} = \frac{1 + \frac{3}{2}P_Y}{2}$$

Plugging these into the resource constraint, we get:

$$Y_{Ann} + Y_{Bob} = 2 \rightarrow \frac{1 + \frac{3}{2}P_Y}{2P_Y} + \frac{2 + P_Y}{4P_Y} = 2$$

$$P_Y = 1$$

Plugging this in, we get:

$$X_{Ann} = \frac{3}{4}, Y_{Ann} = \frac{3}{4}$$

$$X_{Bob} = \frac{5}{4}, Y_{Bob} = \frac{5}{4}$$

- Note that in this case, the allocations are equal! Why? Because they both have the same preferences and value X and Y equally. What will the contract curve be in this picture? It will be the 45 degree line.
- In the previous case, Bob had a really strong preference for X relative to Y and therefore, Y is relatively cheap (in the previous case, P_Y , which is the relative price of Y, is $\frac{1}{2}$, but in this case, the relative price of Y is 1.

7 Go back to the original preferences of the problem. Suppose the government decides that the competitive equilibrium is not a good allocation and they would prefer for Ann to consume $(\frac{3}{4}, \frac{3}{2})$ and Bob to consume $(\frac{5}{4}, \frac{1}{2})$. Is this a competitive equilibrium? Is it attainable from the initial endowment? Why or why not?

- This is a competitive equilibrium as long as it is on the contract curve. We derived above that the contract curve is defined by $Y_{Ann} = 2X_{Ann}$ and we can see that this is the case with this allocation. Therefore, it is a competitive equilibrium.

- If we compare the utilities the Ann and Bob get at this allocation to the utilities that they get at their initial endowments, we can see that Ann is better off and Bob is worse off.

$$U_{Ann}(e_X, e_y) = \frac{1}{2} < \frac{9}{8} = U_{Ann}\left(\frac{3}{4}, \frac{3}{2}\right)$$

$$U_{Bob}(e_X, e_y) = \frac{7}{2} < 3 = U_{Bob}\left(\frac{5}{4}, \frac{1}{2}\right)$$

- However, this equilibrium is not attainable from this original endowment. At the equilibrium prices ($P_Y = \frac{1}{2}$), Ann cannot afford to consume this bundle. Her endowment is worth $\frac{5}{4}$ but the desired bundle costs $\frac{6}{4}$.

8 Suppose that the government can announce that $P_Y = \frac{1}{4}$. Would this achieve the government's desired bundle?

- No, this would not. At the price $P_Y = \frac{1}{4}$, Ann is now able to afford the bundle that the government wants her to consume, since at this price, the new bundle costs $\frac{3}{4} + \frac{3}{2} \cdot \frac{1}{4} = \frac{9}{8}$ and her endowment is also worth $1 + \frac{1}{2} \cdot \frac{1}{4} = \frac{9}{8}$.
- However, this will not be supported in equilibrium. This is because at these prices, Ann and Bob will not want to consume this bundle. We can see this by looking at their demand functions and plugging in $P_Y = \frac{1}{4}$.

$$X_{Ann} = \frac{2 + P_Y}{4} = \frac{9}{16}$$

$$Y_{Ann} = \frac{2 + P_Y}{4P_Y} = \frac{9}{4}$$

- Bob is going to be at the corner of the box - the first order conditions that determine his demand are not satisfied when $P_Y = \frac{1}{4}$ and therefore, he is going to go to a corner at the box. Now that good Y is so cheap, his demand for it is insatiable, so he will demand as much of it as he can (and he will go to the corner with $Y_{Bob} = 2$. Then, the amount of X he can get is determined by his budget constraint, yielding $X_{Bob} = \frac{7}{8}$

9 Could the government achieve its objectives through lump sum redistribution? If so, how should it redistribute to achieve its desired bundle?

- Yes! The government can achieve this allocation through lump sum redistribution. We showed before that all of the points on the contract curve were pareto efficient allocations, and we also showed that this is a bundle on the contract curve. Therefore, the second welfare theorem says that this can be achieved beginning at some endowment.
- What should they redistribute? One possibility is for them to directly redistribute so that the agents start at the desired bundle. They are on the contract curve at this point, so even when they are allowed to trade, they will not (because there are no mutually beneficial trades left). However, you could also move them to any other point on the line that goes through the desired point on the contract curve and has a slope of $\frac{1}{2}$ (i.e. the market equilibrium price)

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