

**14.01 Fall 2010**  
**Problem Set 9 Solutions**

**1. (28 points) R&D and effect of government borrowing**

A pharmaceutical company is considering whether to invest in the research and development of a new drug. It will incur a cost of \$100 million starting in this year for 10 years, and it will get a patent that is worth \$1.5 billion at the beginning of the 11th year. Alternatively to the R&D project, the firm can expand advertising for an already existing drug, which will increase its profits by \$10 million forever. The interest rate in the economy is 5% and is constant over time.

- (a) (7 points) What's the present value of the project? Write an expression for the net present value.

$$PV = \frac{1500}{1.05^{10}} \cdot 10^6 \text{ dollars}$$

When considering whether to invest in the project or not, the firm has to take into account the full opportunity cost of not investing, which amounts to an annual outlay of \$100 million for conducting the R&D plus the forgone increase in profits of \$10 million per year. Hence,

$$NPV = -100 \cdot \left( 1 + \frac{1}{1.05} + \frac{1}{1.05^2} + \dots + \frac{1}{1.05^9} \right) \cdot 10^6 - \left( \frac{10}{0.05} + \frac{1500}{1.05^{10}} \right) \cdot 10^6$$

- (b) (7 points) Suppose that the internal rate of return on the project is 5.28%. Will the project be undertaken? Why?

Since the IRR of the project is higher than the interest rate in the economy, the project will be undertaken.

- (c) (7 points) The government is considering increasing government consumption. Suppose that the market supply of funds is given by  $Q_S = i$ , where  $Q_S$  is funds supplied per year (in billion) and  $i$  is the interest rate. Market demand for funds is given by  $Q_D = 10 - i$ . The government is considering permanently increasing annual borrowing by \$1 billion. Will the project get undertaken now? Why?

By increasing borrowing, the government shifts out the market demand for funds by \$1 billion. Hence, the new equilibrium interest rate will be given by  $Q_S = Q_D + 1$  or  $i = 5.5\%$ . Hence, the IRR now is lower than the interest rate in the economy and the project will not get undertaken.

- (d) (7 points) Given your results in part (c), discuss why a fiscal expansion could hurt productivity growth in the long run.

A fiscal expansion leads to an increase in the equilibrium interest rate in the economy, due to increased demand for funds. This decreases private investment of any kind, in particular investment in R&D as, for example, happens with the project in part (c). Since productivity growth depends on R&D investment, lower R&D investment would hurt growth.

Problem 1 solution courtesy of Plamen Nenov. Used with permission.

**2. (28 points) Intertemporal consumption and savings supply**

Suppose that there are only 10 individuals in the economy, each with the following utility function over present and future consumption:

$$U(c_1, c_2) = c_1 + c_2$$

where  $c_1$  is consumption today, and  $c_2$  is consumption tomorrow. Buying 1 unit of consumption today costs \$1 today, and buying 1 unit of consumption tomorrow costs \$1 tomorrow. All individuals have income of \$10 dollars today and no income tomorrow (they are retired tomorrow), but they can save at the market interest rate  $r \geq 0$ .

- (a) (7 points) What is the price today of one unit of consumption tomorrow? Why?

Given an interest rate of  $r$ , in order to enjoy a unit of consumption tomorrow, an individual has to save  $\frac{1}{1+r}$  dollars of their income today. Hence today's price of a unit of consumption tomorrow is  $p = \frac{1}{1+r}$ .

- (b) (7 points) Write an expression for an individual's budget constraint in terms of today's and tomorrow's consumption expenditure.

The budget constraint will be:

$$c_1 + \frac{1}{1+r}c_2 = 10$$

- (c) (7 points) How much of their income would an individual consume, and how much would they save, given the interest rate of  $r$ ?

We have to solve the utility maximization problem:

$$U(c_1, c_2) = c_1 + c_2$$

such that

$$c_1 + \frac{1}{1+r}c_2 = 10$$

Note that in the context of intertemporal consumption, what is known as the marginal rate of substitution is referred to as the rate of time preferences.

Given the linear preferences, we find that as long as  $r > 0$ , then  $c_1 = 0$  and  $\frac{1}{1+r}c_2 = 10$  or  $c_2 = 10(1+r)$ . For  $r = 0$ , any combination of  $c_1$  and  $c_2$  such that  $c_1 + c_2 = 10$  is optimal.

- (d) (7 points) Suppose that the market demand for funds is given by  $Q_D = 100 - i$ . What is the market supply for funds? What is the equilibrium interest rate that clears the capital market? What is aggregate consumption at that interest rate?

Given the optimal consumption/saving decision of an individual, we have the following individual supply curve:

$$q_S = \begin{cases} 10 & \text{if } r > 0 \\ [0, 10] & \text{if } r = 0 \end{cases}$$

Hence, the market supply of 10 individuals is:

$$Q_S = \begin{cases} 100 & \text{if } r > 0 \\ [0, 100] & \text{if } r = 0 \end{cases}$$

Given the market demand, the equilibrium interest rate that clears the capital market is  $i = 0$  with equilibrium demand of  $Q_D = 100$ . Aggregate consumption is then 0.

Problem 2 solution courtesy of Plamen Nenov. Used with permission.

### 3. (15 points) Demand for flu shots

The demand for flu shots this season is given by:  $P = 13 - 0.0005Q$ . The marginal cost of a flu shot is \$8.

- (a) (5 points) In a competitive market, what are the equilibrium price and quantity of flu shots?  
Set price equal to the (private) marginal cost to get  $P = 13 - 0.0005Q = 8$ , so  $P = 8$  and  $Q = 10,000$ .

- (b) (5 points) The social benefit of flu shots is  $SB = 13Q - 0.0005\frac{Q^2}{4}$ . What is the socially optimal quantity in the market? Compare your result here to the quantity in part (a). Explain any differences you see.

The social benefit is defined to include any private benefits as well here. The social marginal benefit from flu shots is  $MSB = 13 - 0.00025Q$ . Set  $MSB = MC$  to find the socially optimal price and quantity of flu shots.  $13 - 0.00025Q = 8$ ,  $0.00025Q = 5$ ,  $Q = 20,000$ , and from the demand  $P = 3$ .

- (c) (5 points) What government policies could be implemented to achieve the social optimum in this case?

There are different ways to achieve the socially optimal number of flu shots. The government could subsidize flu shots by \$5 each, so the effective price of a flu shot becomes  $8 - 5 = 3$ . Since at

this effective price there is demand for 20,000 flu shots, the socially optimal level of immunization is achieved. The government could alternatively require 20,000 people to get the shot by passing an appropriate law. This works well if we happen to have 20,000 (or fewer) people in the society, but if there are more than 20,000 people, it is not clear which 20,000 should be mandated to get the shot.

4. **(29 points) Government Redistribution and Social Welfare**

Consider an economy with only one good, food. There are three people in the economy,  $A, B$  and  $C$ .  $A$  has 400 units of food,  $B$  has 100 units, and  $C$  has only 16 units. All have the same utility,  $U_i = \sqrt{f}$  for  $i = A, B, C$ . The social welfare function for this society is the sum of the utilities of the three individuals.

- (a) (6 points) If each agent simply consumes their own endowment, what is the utility level for  $A, B$  and  $C$ ? Find the social welfare level.

Utilities are respectively  $U_A = 20, U_B = 10, U_C = 4$ . Welfare is  $W = 34$ .

- (b) (8 points) The government decides to redistribute food more equally, so it takes 175 units from  $A$  and gives them to  $B$ . However, the government spoils 79 of these units in transportation, so  $B$  ultimately gets only 96 units of food. What is each person's utility level now? Find the social welfare level in this case.

$U_A = 15, U_B = 14, U_C = 4, W = 33$ .

- (c) (8 points) Assume now that the government considers a different redistribution scheme. Starting with the original endowments, the government takes 175 units from  $A$ . This time it wishes to give them to  $C$ , but in transportation it destroys 91 units, so  $C$  only gets 84 of these extra units. What is each person's utility level now? Find the social welfare level in this case.

$U_A = 15, U_B = 10, U_C = 10, W = 35$ .

- (d) (7 points) Compare parts (b) and (c) in terms of social welfare. Note that the government is more wasteful in (c), and explain your result.

Since the marginal utility of food is decreasing, in order to increase welfare it is better to transfer food from the richest to the poorest. Hence, even if there is more waste in transferring resources from  $A$  to  $C$  than from  $A$  to  $B$ , when such a difference in waste is relatively small, then welfare increases more when food is transferred to those that have a higher marginal value for it.

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